Section 4.8 Antiderivatives

A function F(x) is an antiderivative of the function f(x) if F'(x) = f(x).
For example the function F(x) = $x^2$ is an antiderivative of f(x)=2x .
$x^2$+1, $x^2$+2 are also antiderivatives of 2x as is $x^2$+C for any constant C.

$x^2$+C is called the most general antiderivative of 2x.

Reversing the Power rule: For n not equal to -1,
$\frac{d}{dx}(\frac{1}{n+1}x^{n+1}) = \frac{1}{n+1}(n+1)x^n = x^n$ *so* $\frac{1}{n+1}x^{n+1} + C$ *is the most general antiderivative of* $x^n$.

| function | most general antiderivative |
|---|---|
| $x^n, n \neq -1$ | $\frac{1}{n+1}x^{n+1} + C$ |
| $\frac{1}{x} = x^{-1}$ | $\ln\|x\| + C$ |
| $e^x$ | $e^x + C$ |
| $\sin x$ | $-\cos x + C$ |
| $\cos x$ | $\sin x + C$ |
| $\sec^2 x$ | $\tan x + C$ |
| $\sec x \tan x$ | $\sec x + C$ |

There is no product rule or quotient rule for antiderivatives. The linear rule and the shift rule work just as they do for derivatives.

Examples: Find the most general antiderivative, F, of each function, f.

1. $f(x) = x^2$ $F(x) = \frac{1}{3}x^3 + C$ 2. $f(x) = 5x^2$ $F(x) = \frac{5}{3}x^3 + C$
3. $f(x) = x^{1/2}$ $F(x) = \frac{2}{3}x^{3/2} + C$ 4. $f(x) = 4x^{1/2}$ $F(x) = \frac{8}{3}x^{3/2} + C$
5. $f(x) = 5x^2 + 4x^{1/2}$ $F(x) = \frac{5}{3}x^3 + \frac{8}{3}x^{3/2} + C$
6. $f(x) = 4e^x + \sin x + 3$ $F(x) = 4e^x - \cos x + 3x + C$

Example: Find the most general antiderivative of $(x+1)\sqrt{x}$ .
The antiderivative of the product is not the product of the antiderivatives.
We must multiply it out.

$(x+1)\sqrt{x} = x\sqrt{x} + \sqrt{x} = x^{3/2} + x^{1/2}$ $F(x) = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$

Example: Find the most general antiderivative of $x\sqrt{x+1}$.

$$x\sqrt{x+1} = (x+1-1)\sqrt{x+1} = (x+1)\sqrt{x+1} - \sqrt{x+1} = (x+1)^{3/2} - (x+1)^{1/2}$$

$$F(x) = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

Finding a specific antiderivative that meets a given condition:

Example 1. Find an antiderivative, *F(x)*, for $f(x) = \dfrac{x^2+2x+3}{x}$ *so* $F(1) = 0$.

There is no quotient rule for antiderivatives. The antiderivative is not the quotient of the antiderivatives. We must divide.

$f(x) = x + 2 + \dfrac{3}{x}$ $\quad F(x) = \dfrac{1}{2}x^2 + 2x + 3\ln x + C$ is the most general antiderivative.

Now plug in *x*=1 and solve for *C*. F(1) = 0.5 + 2 +0 +C = 0 so C= -2.5

$$F(x) = \frac{1}{2}x^2 + 2x + 3\ln x - 2.5$$

Example 2. An object is fired straight up from a height of 6 m with initial velocity 20 m/sec. Given the acceleration due to gravity is -9.8 m/sec^2, find the height at time t seconds.

$$a(t) = -9.8 \quad v(t) = -9.8t + C \quad \text{and} \quad v(0) = 20 \quad \text{so} \quad v(t) = -9.8t + 20$$

$$h(t) = -9.8(\frac{1}{2}t^2) + 20t + C \quad \text{and} \quad h(t) = 6 \quad \text{so} \quad h(t) = -4.9t^2 + 20t + 6$$

Example 3. If the acceleration of an object moving in a straight line is given by

$a(t) = 2e^t + 1$ *and* $v(0) = 4$ *and* $s(0) = 10$ , find *s(t)*.

$$v(t) = 2e^t + t + C \quad v(0) = 2 + 0 + C = 4 \text{ so } C = 2 \quad v(t) = 2e^t + t + 2$$

$$s(t) = 2e^t + \frac{1}{2}t^2 + 2t + C \quad s(0) = 2 + C = 10 \text{ so } C = 8$$

$$s(t) = 2e^t + \frac{1}{2}t^2 + 2t + 8$$