Section 5.1 – Simple Interest, Future Value, Present Value, and Effective Rate

Simple Interest is interest that is compounded on the original principal only.

P = principal

r = interest rate per year (% to decimal)

t = time (in years)

Accumulated Amount is the sum of the principal and interest after t years.

Formula: $A = P + I$

$A = P + Prt$

$A = P(1+rt)$

Example 1: Find the simple interest on a $1000 investment made for 3 years at an interest rate of 5% per year. What is the accumulated amount?

$A = P(1+rt)$

$A = 1000(1 + .05(3))$

$A = \$1150.00$

Example 2: Find the simple interest rate at which $1,000 will grow to $1,050.

in nine months.

$A = 1{,}050$

$P = 1{,}000$

$t = 9/12 = 3/4$

$A = P(1+rt)$

$1050 = 1000(1 + r(3/4))$

$1.05 = 1 + r(3/4)$

$.05 = r(3/4)$

$3/4(.05) = r(3/4)(4/3)$

$r = .06666$

$r = .067$ or 6.7%

Compounded Interest is earned interest that is periodically added to the principal and there after earns interest at the same rate.

$A = P(1+i)^n$ $\quad i = \frac{r}{m}$ $\quad n = m \cdot t$

A stands for the future value or the accumulated amount at the end of n conversion periods. A conversion period refers to the interval of time between successive interest calculations.

P stands for the present value or principal.

r stands for the interest rate per year.

m stands for the number of compounding periods per year.

t stands for time (years).

 Fv of Compound interest

Example 3: Find the accumulated amount after 5 years if $1700 is invested at 6.25% per year compounded

$A = P(1+i)^n$

a. quarterly. $m = 4$

$P = 1700$

$r = .0625$

$n = m \cdot t = 4(5) = 20$

$A = 1700\left(1 + \frac{.0625}{4}\right)^{20}$

$A = \$2318.02$

b. semiannually. m_2

$i = \frac{r}{m} = \frac{.0625}{2}$

$n = m \cdot t = 2(5) = 10$

$A = 1700\left(1 + \frac{.0625}{2}\right)^{10}$

$A = \$2312.54$

Present Value

Recall $A = P(1 + i)^n$ and P = Present Value
Why would we want to find P?

Well in certain instances an investor may wish to determine how much money he should invest now, at a fixed rate of interest, so that he/she will realize a certain sum of money at some future date.

So, solving the Future Value formula for P we obtain the Present Value with compound interest formula.

$$A = P\left(1 + \frac{r}{m}\right)^n \Rightarrow P = \frac{A}{\left(1 + \frac{r}{m}\right)^n} \Rightarrow P = A\left(1 + \frac{r}{m}\right)^{-n}$$

Example 4: Kim and Ken find that they will need $15,500 to build an addition to their home in 4 years. How much should they invest now at 3.25% per year compounded quarterly to have the desired funds in 4 years?

$P = A(1+i)^{-n}$

$A = 15,500$

$r = .0325$

$m = 4$

$n = m \cdot t = 4(4) = 16$

$P = 15,500\left(1 + \frac{.0325}{4}\right)^{-16}$

$P = \$13,617.63$

Example 5: A newborn child receives a $5000 gift towards a college education from her grandparents. How much will the $5000 be worth in 17 years if it is invested at 9% per year compounded quarterly?

$A = P(1+i)^n$

$P = 5{,}000$

$r = .09$

$m = 4$

$n = m \cdot t = 4(17) = 68$

$A = 5{,}000\left(1 + \frac{.09}{4}\right)^{68}$

$A = \$22{,}702.60$

Example 6: An Individual Retirement Account (IRA) has $20,000 in it and the owner decides not to add any more money to the account other than interest earned at 8% per year compounded monthly. How much will be in the account 35 years from now when the owner reaches retirement age?

$A = P(1+i)^n$

$P = 20{,}000$

$r = .08$

$m = 12$

$n = m \cdot t = 12(35) = 420$

$A = 20{,}000\left(1 + \frac{.08}{12}\right)^{420}$

$A = \$325{,}851.00$

Example 7: Kim invested a sum of money 4 years ago in a savings account that has since paid interest at the rate of 6.5% per year compounded monthly. Her investment is now worth $19,440.31. How much did she originally invest?

$P = A(1+i)^{-n}$

$A = 19{,}440.31$

$r = .065$

$m = 12$

$n = m \cdot t = 12(4) = 48$

$P = 19{,}440.31\left(1 + \frac{.065}{12}\right)^{-48}$

$P = \$15{,}000.00$