Section 5.2 $\int_a^b f(x)dx$, the integral from a to b of *f(x)* .

For any **continuous** function, *f(x)*, $\int_a^b f(x)dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$ where L_n is the left hand sum and R_n is the right hand sum for n equal subintervals of [a, b].

The general definition of integrable and the general Riemann sum are in the text. The intervals need not be equal and any function value in a subinterval can be the "height" for the rectangle. **If the function is negative in the subinterval, the contribution from that subinterval is negative**. We will use functions which are continuous but not necessarily non-negative but will not use general Riemann sums. We will use L_n , R_n , and their average.

Example: Find L_n , R_n , and their average for $f(x) = x^3$ on [-1, 2] with *n*=6.

The partition of [-1, 2] has subinterval lengths $\frac{b-a}{n} = \frac{2-(-1)}{6} = \frac{1}{2}$ We start with -1 and add increments of 1/2 until we get to 2.

x	-1	$-1/2$	0	$1/2$	1	$3/2$	2
$f(x)$	-1	$-1/8$	0	$1/8$	1	$27/8$	8

For L_n we use all but the last, for R_n we use all but the first of these values in the function.

L_6 = (1/2)(-1) + (1/2)(-1/8) + (1/2)(0) + (1/2)(1/8) + (1/2)(1) + (1/2)(27/8)
= (1/2)[-1 + -1/8 + 0 + 1/8 + 1 + 27/8]= 27/16
Similarly,
R_6 = (1/2)[-1/8 +0 + 1/8 +1 + 27/8 + 8] = 99/16

$$\frac{L_6 + R_6}{2} = \frac{126}{32} = \frac{63}{16} = 3.9375$$

Properties of $\int_a^b f(x)dx$:

$\int_a^b f(x)dx =$ the Area Above the x-axis under f minus the Area Below the x-axis above f

$$\int_a^b Af(x) + Bg(x)dx = A\int_a^b f(x)dx + B\int_a^b g(x)dx$$

$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$