Section 5.5 Substitution in Integrals

Substitution type I is reversing the chain rule.
Substitution type II is making a linear substitution so powers can be multiplied
They are done the same way but recognized differently in the integrand.

I. Type I. Recall the Chain Rule:

$$\frac{d}{dx}\left[f(u(x))\right] = f'(u(x))\, u'(x)$$

Ex. 1 $\frac{d}{dx}\left[(x^2+1)^3\right] = 3(x^2+1)^2 \cdot 2x$

Here, $u(x) = x^2+1$, the inside of the composite. Going the other way:

$$(x^2+1)^3 + C = \int 3(x^2+1)^2\, 2x\, dx$$

You will have the integral and have to find the anti-derivative. So you see a product and one part is a composite, $f(u(x))$; the other part is $u'(x)$, the derivative of the inside of the composite.

Ex 2. $\int e^{x^2} \cdot 2x\,dx = \int e^{u(x)}\, u'(x)\,dx$, $u(x)=x^2$.

$= e^{u(x)} + C = e^{x^2} + C$

We write this integral as:

$$\int \underbrace{e^{x^2}}_{e^u} \underbrace{2x\,dx}_{du} = \int e^u\,du = e^u + C = e^{x^2} + C$$

That is: $u = x^2$
$du = u'(x)\,dx = 2x\,dx$

Process for Substitution I:

Recognize the composite $f(u(x))$.
Let $u = u(x)$
$du = u'(x)\,dx$

Replace the integrand with $f(u)\,du$.
Evaluate $\int f(u)\,du$.

Replace all u's with $u(x)$'s

Ex. 3. $\int (e^x+1)^3 \cdot e^x\,dx$

$u = e^x + 1$
$du = e^x\,dx$

$\int u^3\,du = \frac{1}{4}u^4 + C$

$= \frac{1}{4}(e^x+1)^4 + C$

Check by differentiating. You should get the integrand as the derivative of your final answer.

Ex.4. $\int \sqrt{x^3+5x}\,(3x^2+5)\,dx$

$u = x^3 + 5x$

$du = (3x^2 + 5)\,dx$

$$\int \sqrt{u}\,du = \int u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(x^3+5x)^{3/2} + C$$

Ex.5. $\int \frac{(\ln x)^2}{x}\,dx = \int (\ln x)^2 \cdot \frac{1}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int u^2\,du = \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}(\ln x)^3 + C$$

Ex.6. $\int e^{2x^3+7x}(6x^2+7)\,dx$

$u = 2x^3 + 7x$

$du = (6x^2+7)\,dx$

$$\int e^u\,du = e^u + C$$

$$= e^{2x^3+7x} + C$$