Section 7. 1 Nov. 2001

Nonparametric methods of uncertainty estimination

δ-method, (infinitesimal) jackknife, bootstrap, cross-validation

Techniques broadly applicable, complex statistics
Generally justified by asymptotics
There exist singular and inappropriate cases
NECESSARY

δ-method AKA method of linearization, propagation of error, Taylor series method

Gauss (1815)

Basically one approximates functions by Taylor expansions (usually linear) of basic random variables.

Rao, Section 6a.2

Sequence of k-dimensional statistics

$$\underset{\sim}{T}_n = (T_{1n}, \dots, T_{kn}) \qquad n = 1, 2, \dots$$

eg. $(\bar{X}, \bar{Y})$

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Derived statistic

$$g(\underset{\sim}{T}_n) = g(T_{1n}, \ldots, T_{kn})$$

eg. $g(\bar{X}, \bar{Y}) = \dfrac{\bar{Y}}{\bar{X}}$

Suppose $\boxed{\sqrt{n}\,(\underset{\sim}{T}_n - \underset{\sim}{\theta}) \xrightarrow{d} N_k(0, \underset{\sim}{\Sigma})}$ $\xrightarrow{L}$

Write

$$g(\underset{\sim}{T}_n) = g(\underset{\sim}{\theta}) + \frac{\partial g(\underset{\sim}{\theta})}{\partial \underset{\sim}{\theta}} \cdot (\underset{\sim}{T}_n - \theta) + \ldots$$

The entity of principal interest is typically $g(\underset{\sim}{\theta})$

Theorem. If $g(\,)$ has a continuous first derivative

$$\boxed{\sqrt{n}\,\{g(\underset{\sim}{T}_n) - g(\theta)\} \xrightarrow{d} N\left(\underset{\sim}{0}, \frac{\partial g}{\partial \theta}^{T} \underset{\sim}{\Sigma} \frac{\partial g}{\partial \theta}\right)}$$

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Convergence in distribution (law)

Sequence of r.v.'s $\{X_n\}$

$$F_n(x) = \text{Prob}\{X_n \le x\}$$

$X_n \xrightarrow{d} X$ $\quad (X_n \xrightarrow{L} X)$ $\quad$ (weakly)

if $F_n(x) \to F(x)$ at all continuity points of F

equivalent to

$\int g \, dF_n \to \int g \, dF$ $\quad$ for all bounded continuous g

Doesn't always hold if g is unbounded.

e.g. $X_n = \mu + \sigma Z + \frac{1}{n} C$ $\quad$ C: Cauchy

$$X_n \xrightarrow{d} X \sim N(\mu, \sigma^2)$$

$$EX_n = \infty \;, \quad EX = \mu$$

$\text{var}\, h(X_n) \not\to \text{var}\, h(X)$ $\quad$ generally