Section 5.3 Evaluating Integrals

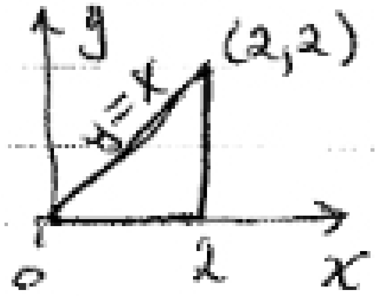
FTC = Fundamental Theorem of Calculus

If $f(x)$ is continuous on $[a, b]$ and
if $F'(x) = f(x)$ on $[a, b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

The notation used for $F(b) - F(a)$

Examples:



1) We can use geometry to find $\int_0^2 x\,dx$

Since $y = x$ is non-negative $\int_0^2 x\,dx = \text{area} = \frac{1}{2}(2\times 2)$
$= 2$

By FTC, $\int_0^2 x\,dx = \frac{1}{2}x^2\Big|_0^2 = \frac{1}{2}(2^2) - \frac{1}{2}(0^2) = 2$

2) $\int_1^2 7x + 8\,dx = 7\left(\frac{1}{2}x^2\right) + 8x\Big|_1^2$

$= \frac{7}{2}x^2 + 8x\Big|_1^2 = \frac{7}{2}(2^2) + 8(2) - \left[\frac{7}{2}(1^2) + 8\right]$

$= 14 + 16 - \left(\frac{7}{2} + 8\right)$

$= \frac{37}{2}$

3) $\int_1^8 \frac{1}{\sqrt[3]{x}}\,dx = \int_1^8 x^{-1/3}\,dx = \frac{3}{2}x^{2/3}\Big|_1^8$

$= \frac{3}{2}\cdot 8^{2/3} - \frac{3}{2}\cdot 1^{2/3} = 6 - 3/2 = 9/2$

Warning: What is wrong here?

The integral over $[a, b]$ of a non-negative function is the area under the graph over the interval $[a, b]$.

$\frac{1}{x^2} > 0$

$$\int_{-1}^{1} \frac{1}{x^2}\, dx = \int_{-1}^{1} x^{-2}\, dx = -x^{-1} \Big|_{-1}^{1} = \left(-1 - \underbrace{\left(-(-1)^{-1}\right)}_{1}\right)$$

$$= -1 - 1 = -2$$

Why is it negative? To be answered in class.

Some integrals are still done with geometry.

$\int_0^1 \sqrt{1-x^2}\, dx$ is $\frac{1}{4}$ of the area of the unit circle.

$$\int_0^1 \sqrt{1-x^2}\, dx = \frac{1}{4}\pi$$

More Examples:

4) $\int_0^1 x^{1/3} + e^x\, dx = \frac{3}{4}x^{4/3} + e^x \Big|_0^1$

$= \frac{3}{4} 1^{4/3} + e^1 - (0 + e^0) = 3/4 + e - 1 = e - 1/4$

5) $\displaystyle\int_0^{\pi/2} \sin x\, dx = -\cos x \Big|_0^{\pi/2} = \overset{0}{-\cos \tfrac{\pi}{2}} \overset{-(-1)}{-(-\cos 0)}$

$= 1$

6) $\displaystyle\int_1^4 \frac{x^2+2x-7}{x}\, dx = \int_1^4 x + 2 - \frac{7}{x}\, dx$

$\displaystyle = \frac{1}{2}x^2 + 2x - 7\ln x \Big|_1^4 = \frac{1}{2}\cdot 4^2 + 2\cdot 4 - 7\ln 4$

$\displaystyle - \Big(\underset{1/2}{\tfrac{1}{2}\cdot 1^2} \underset{+2}{+ 2\cdot 1} \underset{-0}{- 7\ln 1}\Big)$

$\displaystyle = 8 + 8 - 7\ln 4 - \left(\tfrac{5}{2}\right) = \frac{27}{2} - 7\ln 4$

7) $\displaystyle\int_{-2}^{-3} (x+1)(x-2)\, dx = \int_{-2}^{-3} (x^2 - x - 2)\, dx$

Note: $-2 > -3$
but don't put it
on top unless you
negate the integra

$\displaystyle = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x \Big|_{-2}^{-3}$

$\displaystyle = \frac{1}{3}(-27) - \frac{1}{2}(-3)^2 - 2(-3) - \Big[\frac{1}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 2(-2)\Big]$

$\displaystyle = -9 + \frac{9}{2} + 6 - \left(-\frac{8}{3} - 2 + 4\right)$

$\displaystyle = -5 + \frac{9}{2} + \frac{8}{3} = \frac{-30 + 27 + 16}{6} = \frac{13}{6}$