

ME451: Control Systems

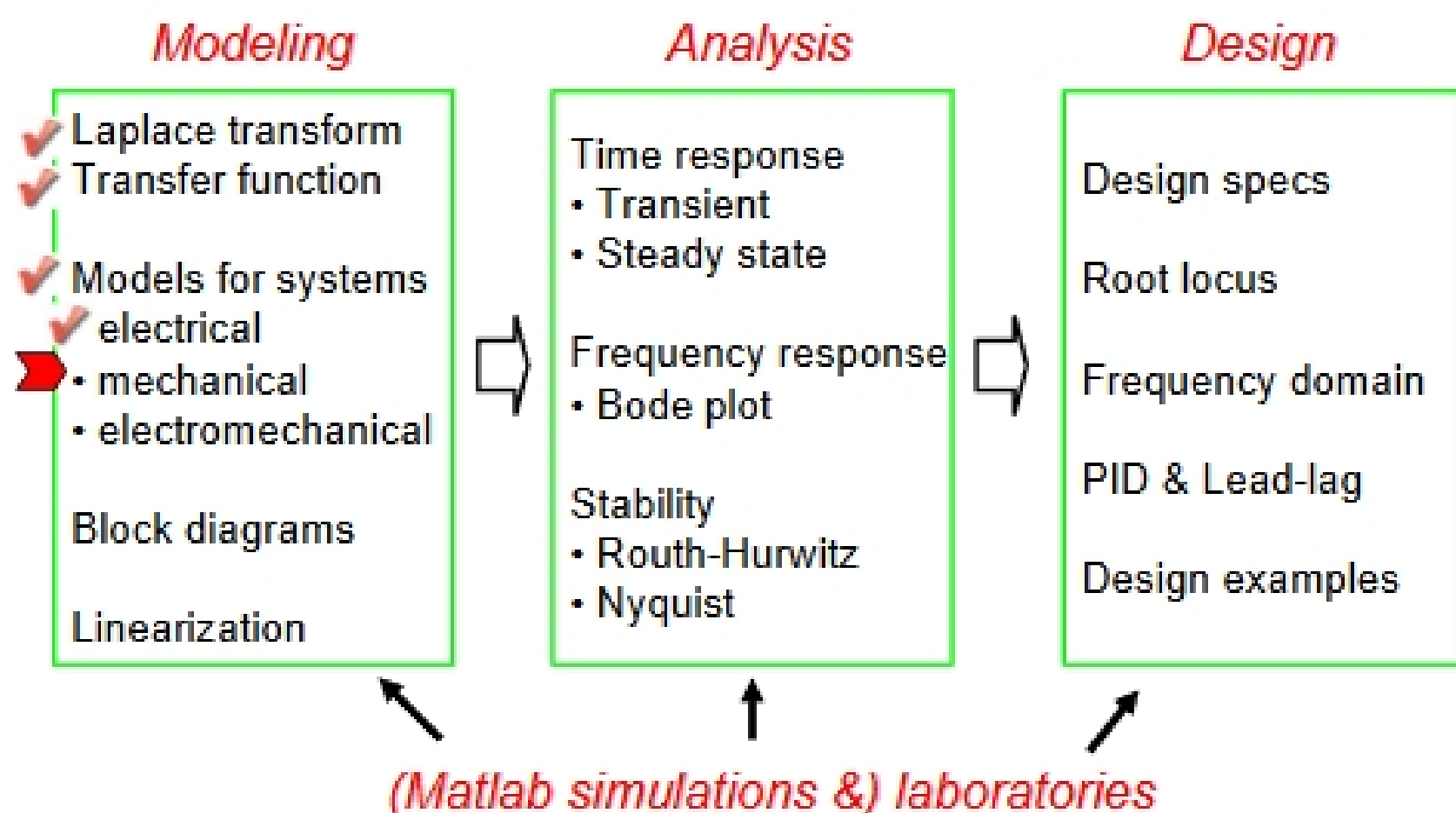
Lecture 5 Modeling of mechanical systems

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Fall 2008

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Course roadmap

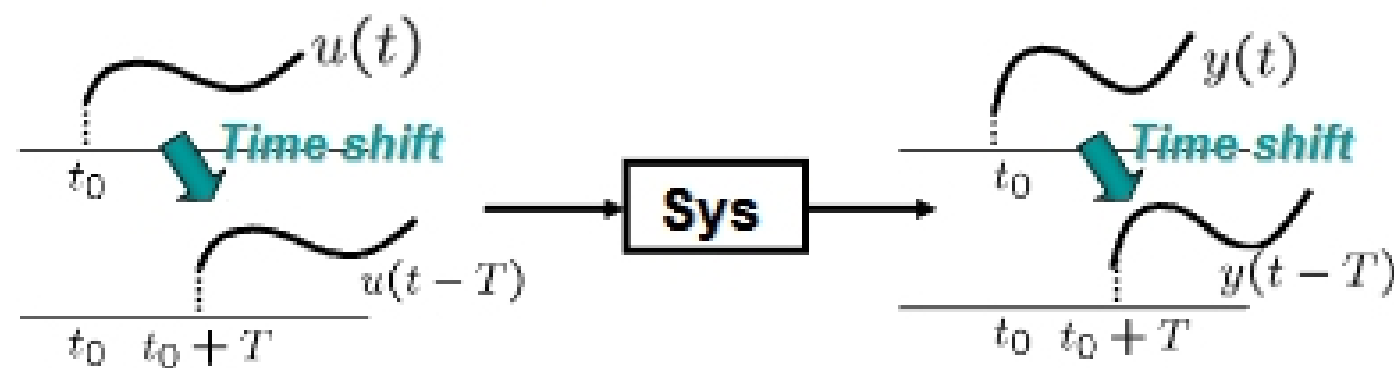


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Time-invariant & time-varying

- A system is called **time-invariant (time-varying)** if system parameters do not (do) change in time.
- Example: $Mx''(t)=f(t)$ & $M(t)x''(t)=f(t)$
- For time-invariant systems:



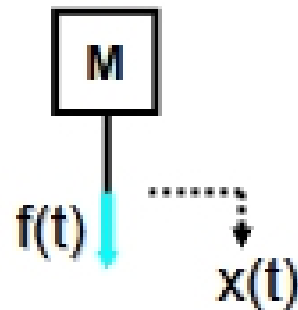
- This course deals with time-invariant systems.

Newton's laws of motion

- **1st law:**
 - A particle remains at rest or continues to move in a straight line with a constant velocity if there is no unbalancing force acting on it.
- **2nd law:**
 - $\sum F_i(t) = m \frac{d^2x}{dt^2}$: translational
 - $\sum \tau_i(t) = I \frac{d^2\theta}{dt^2}$: rotational
- **3rd law:**
 - For every action has an equal and opposite reaction

Translational mechanical elements: (constitutive equations)

Mass

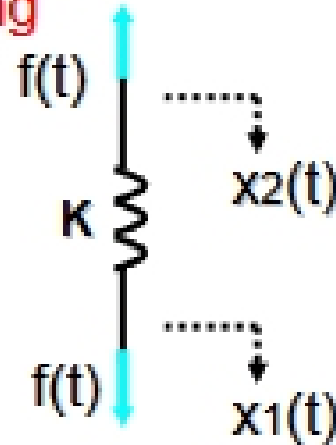


$$f(t) = Mx''(t)$$

$$\downarrow \begin{pmatrix} x(0) = 0 \\ \dot{x}(0) = 0 \end{pmatrix}$$

$$F(s) = Ms^2X(s)$$

Spring

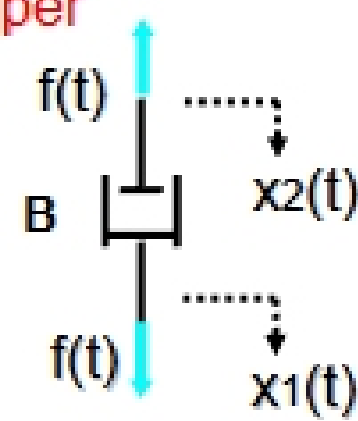


$$f(t) = K(x_1(t) - x_2(t))$$

$$\downarrow$$

$$F(s) = K(X_1(s) - X_2(s))$$

Damper



$$f(t) = B(x_1'(t) - x_2'(t))$$

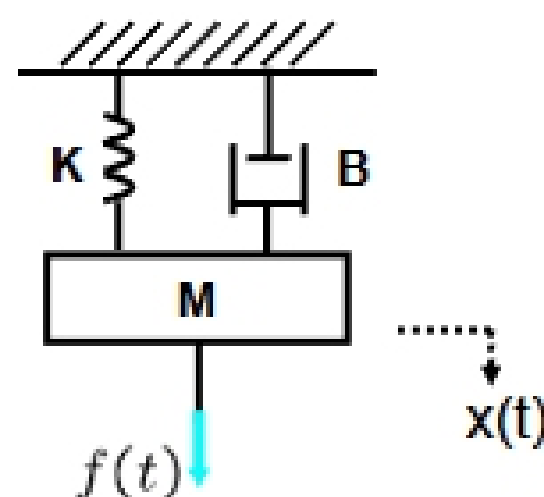
$$\downarrow \begin{pmatrix} x_1(0) = 0 \\ x_2(0) = 0 \end{pmatrix}$$

$$F(s) = Bs(X_1(s) - X_2(s))$$

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Mass-spring-damper system



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