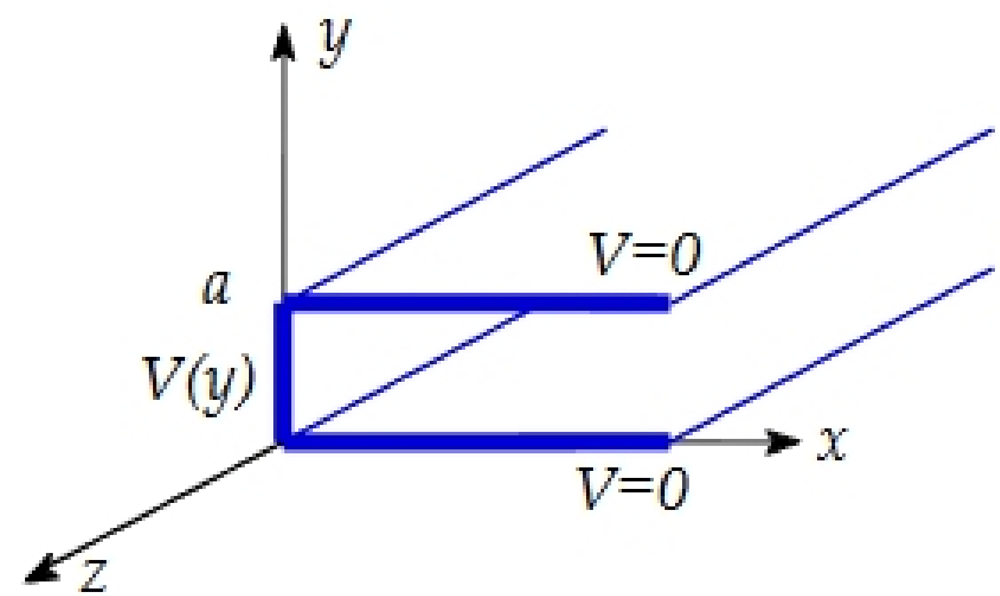
# Today in Physics 217: separation of variables

- Introduction to the method, in Cartesian coordinates.
- Example solution for the potential in an infinite slot, arbitrary $V$ at the bottom, in which we introduce two common features of separation solutions:
  - Completeness and orthogonality of sines
  - Fourier's trick



# Introduction to separation of variables

- If the method of images can't be used on a problem you must solve that involves conductors, the next thing to try is direct solution of the Laplace equation, subject to boundary conditions on $V$ and/or $\partial V/\partial n$.
- Separation of variables is the easiest direct solution technique. It works best with conductors for which the surfaces are well behaved (planes, spheres, cylinders, *etc*.).

Here's how it works, in Cartesian coordinates, in which the Laplace equation is

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

which can't be integrated directly like the 1-D case.

# Introduction to separation of variables (continued)

Consider solutions of the form

$$V(x,y,z) = X(x)Y(y)Z(z)$$

Then, the Laplace equation is

$$YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} = 0$$

or, dividing through by $XYZ$,

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

i.e.

$$f(x) + g(y) + h(z) = 0$$