Chapter 3: Oscillations in one dimension

Widely applicable because most situations are boring: slightly displaced from stable equilibrium.

Maclaurin (Taylor) series expansion for potential energy:

$$U(x) = U_0 + U_1 x + \frac{1}{2} U_2 x^2 + \dots + \frac{1}{n!} U_n x^n + \dots$$

$$U_n \equiv \left. \frac{d^n U}{dx^n} \right|_{x=0}$$

Choose $U(x) = 0$ at $x = 0$, so $U_0 = 0$

Equilibrium at $x = 0 \Rightarrow U_1 = 0$

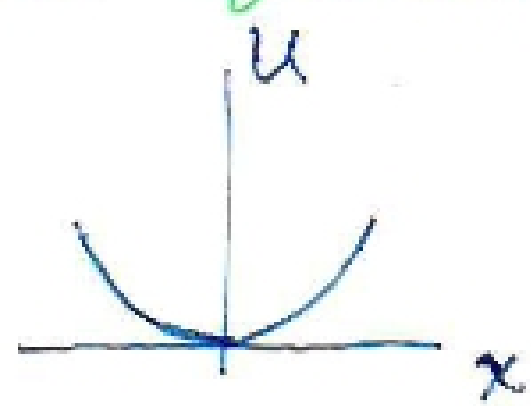
Then, for small $|x|$, $U(x) \doteq \frac{1}{2} U_2 x^2$

$$F = -\frac{dU}{dx} = -U_2 x \qquad (k \equiv U_2)$$

$$\boxed{F = -kx} \qquad \text{Hooke's Law}$$

Pertains to any conservative force near a point of stable equilibrium.

$$U = \frac{1}{2} k x^2$$



$$F = -kx \iff m\ddot{x} = -kx \iff \ddot{x} + \frac{k}{m}x = 0$$

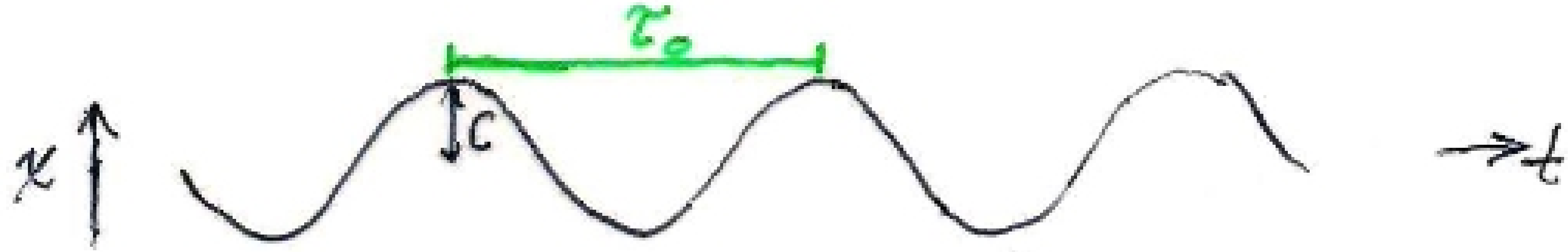
$$\iff \boxed{\ddot{x} + \omega_0^2 x = 0} \qquad \omega_0 \equiv \sqrt{\frac{k}{m}}$$

Two simple solutions: $x = \cos(\omega_0 t)$, $x = \sin(\omega_0 t)$

General solution: $x = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

Match initial conditions: $A = x(0)$, $B = \frac{1}{\omega_0}\dot{x}(0)$.

Equivalent form: $x = C\cos(\omega_0 t - \delta)$ (See next slide)



"Simple harmonic motion"

τ_0 = period (seconds)

$\nu_0 \equiv \frac{1}{\tau_0}$ = frequency (cycles/second)

$\omega_0 \equiv 2\pi\nu_0$ = angular frequency (radians/s)

$E = \frac{1}{2}kC^2$ (potential energy at turnaround points where kinetic energy is 0)

Alternative forms

$$A\cos\theta + B\sin\theta = C\cos(\theta-\varphi)$$
$$\equiv \underline{C\cos\varphi}\,\cos\theta + \underline{C\sin\varphi}\,\sin\theta$$

If C and φ are known, find A and B:

$$A = C\cos\varphi \qquad B = C\sin\varphi$$

If A and B are known, find C and δ:

$$C = +\sqrt{A^2+B^2} \qquad (\cos\varphi, \sin\varphi) = \left(\frac{A}{C}, \frac{B}{C}\right)$$