$$y = \left(\frac{-9-2\sqrt{7}}{-2\sqrt{7}}\right)e^{(-2+\sqrt{7})x} + \left(\frac{9-2\sqrt{7}}{-2\sqrt{7}}\right)e^{(-2-\sqrt{7})x}$$

6-3-1# Imaginary Numbers

$y'' + 2y' + 2y = 0$

$r^2 + 2r + 2 = 0$

$r = \frac{-2 \pm \sqrt{4-4(2)}}{2}$ $r = \frac{-2 \pm \sqrt{-4}}{2}$

$r = -1 \pm i$

$y_1 = C_1 e^{-1+i}$

$= C_1 e^{-1x+ix} = C_1 e^{-x} e^{ix}$

$y_2 = C_2 e^{-1-i}$ $C_2 e^{-x-ix} = C_2 e^{-x} e^{-ix} = C_2 e^{-x}(\cos x - i \sin x)$

$e^{ix} = \cos x + i \sin x$

$e^{\pi} = -1$ $i^2 = -1$

$\boxed{e^{i\pi} + 1 = 0}$

General solution is

$y = e^{-x}\left[\overbrace{(C_1 + C_2)}^{C_1}\cos x + i\overbrace{(C_1 - C_2)}^{C_2}\sin x\right]$

$y = e^{-x}(C_1 \cos x + iC_2 \sin x)$ $y(0) = 3$, $y'(0) = 4$

$3 = C_1(1) + iC_2(0)$ $3 = C_1$

$y' = -e^{-x}(C_1 \cos x + iC_2 \sin x) + e^{-x}(-C_1 \sin x + iC_2 \cos x)$

$4 = -1(C_1(1) + 0) + 1(-C_1(0) + iC_2(1))$

$4 = -C_1 + iC_2$ $7 = iC_2$

$C_2 = 7/i = -7i$

$\boxed{y = e^{-x}(3\cos x + 7\sin x)}$

$y'' + 4y = 0$

$y_0 = 4$

$y_0' = 1$

$r^2 + 4 = 0$

$r = \pm\sqrt{-4}$ $r = \pm 2i$

$y_1 = C_1 e^{2ix} = C_1 e^{i2x} = C_1(\cos 2x + i \sin 2x)$

$y_2 = C_2 e^{-2ix} = C_2 e^{-i2x} = C_2(\cos 2x - i \sin 2x)$

$y = C_1 \cos 2x + C_2 i \sin 2x$ $4 = C_1 + 0$ $C_1 = 4$

$y' = -2C_1 \sin 2x + 2C_2 i \cos 2x$ $1 = -2(0) + 2iC_2(1)$ $C_2 = 1/2i$

$\boxed{y = 4\cos 2x + 1/2 \sin 2x}$

HW Due Tomorrow

3.2 1-5, 7-10, 13, 14, 17, 18, 20, 23

HW Due Thursday

3.3 1-4, 7-12, 17-20