

Linear Algebra 2270-2

Due in Week 3

The third week finishes the first half of the work from chapter 2. Here's the list of problems, followed by problem notes and a few answers.

Section 2.1. Exercises 9, 10, 21, 29, 34

Section 2.2. Exercises 5, 9, 12, 13, 32

Section 2.3. Exercises 3, 13, 19, 25, 27, 28

Problem Notes

Resolution of issues for Strang's problems will appear here in response to communications. If there is a difficulty, like a missing definition or incomplete information, then please write email, call 581-6879, or visit JWB 113.

2.1-29. A matrix A is Markov if each column sum is one and the entries are positive. It is known that $A\vec{x}$ has positive components that add to one, provided the same is true for \vec{x} . An important example is the market share matrix A for three telephone companies ATT, MCI, SPRINT. It helps to solve the problem by maple assist.

Some Answers

2.1. Exercises 29, 34 have answers in the textbook. Reproduced here.

2.1-9. (a) $Ax = (18, 5, 0)$ and (b) $Ax = (3, 4, 5, 5)$.

2.1-10. Multiplying as linear combinations of the columns gives the same Ax . By rows or by columns: 9 separate multiplications for 3 by 3.

2.1-21. $R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. The entries are sines and cosines of 45 degrees.

2.1-29. $u_2 = \begin{pmatrix} .7 \\ .3 \end{pmatrix}$ and $u_3 = \begin{pmatrix} .65 \\ .35 \end{pmatrix}$. The components add to 1. They are always positive. u_7, v_7, w_7 are all close to $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$. Their components still add to 1.

2.1-34. $A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$ and $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ has solution $\vec{x} = \begin{pmatrix} 4 \\ 7 \\ 8 \\ 6 \end{pmatrix}$.

2.2. Exercises 5, 9, 12, 13, 32 do not have textbook answers. Below for answers.

2.2-5. $6x + 4y$ is 2 times $3x + 2y$. There is no solution unless the right side is $2 \times 10 = 20$. Then all the points on the line $3x + 2y = 10$ are solutions, including $(0, 5)$ and $(4, -1)$. (The two lines in the row picture are the same line, containing all solutions).

2.2-9. On the left side, $6x - 4y$ is 2 times $3x - 2y$. Therefore we need $b_2 = 2b_1$ on the right side. Then there will be infinitely many solutions (two parallel lines become one single line).

2.2-12. Elimination first gives an upper triangular system

$$\begin{aligned}2x + 3y + z &= 8 \\ y + 3z &= 4 \\ 8z &= 8\end{aligned}$$

Back-substitution sorts out the answer.

2.2-13. Whatever the elimination details, $x = 3$, $y = 1$, $z = 0$.

2.2-32. The question deals with 100 equations $Ax = 0$ when A is singular. (a) Some linear combination of the 100 rows is the row of 100 zeros.

(b) Some linear combination of the 100 columns is the column of zeros.

(c) A very singular matrix has all ones: $A = \text{eye}(100)$. A better example has 99 random rows (or the numbers $1^i, \dots, 100^i$ in those rows). The 100th row could be the sum of the first 99 rows (or any other combination of those rows with no zeros).

(d) The row picture has 100 planes meeting along a common line through 0. The column picture has 100 vectors all in the same 99-dimensional hyperplane.

2.3. Exercises 3, 25, 27, 28 have a textbook answer.

2.3-13. (a) E times the third column of B is the third column of EB . A column that starts at zero will stay at zero. (b) E could add row 2 to row 3 to change a zero row to a nonzero row.

2.3-19. Any swap elementary matrix S satisfies $S^2 = I$.