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Sets and Operations on Sets

What is a set?

- A **set** is a collection of objects.
- An object in a set is called an **element**.
- Examples: Colors in the rainbow, people in the class, animals in the zoo. The numbers 1,2,3,.... The letters in the alphabet.
- When we talk about sets we must define the **universe**. The universe is all the objects that are allowed to be considered. For example, if P is the set of people in the class, the universe is the set of all possible people in the world.
- **Well defined:** It only makes sense to talk about a set when both (a) there is universe for the set and (b) every object in the universe is either in the set or not in the set. Example: Is the collection of "Best Songs of the 1980's" a set? How can you change the definition to make it a set?

How to describe a set.

1. Verbal description: "The set of people on student government."
2. Listing in braces: {monkey, giraffe, kangaroo, meerkat.}.
3. Set builder notation: $\{x|\ x$is a letter in the alphabet$\}$.

Note on listing sets: The order in which we list the elements in a set *does not matter*. So the set $\{x, y, z\}$ is the same as the set $\{z, y, x\}$. Also, we only list each element of a set once so the sets $\{1, 1, 1, 2, 3\}$ and $\{1, 2, 3\}$ are the same.

Venn Diagrams You can use Venn diagrams to model sets.

Some more definitions for sets.

- If we want to say x is an element of a set A, we will typically write $x \in A$ where the symbol "$\in$" means "in".
- A **natural number** or **counting number** is a member of the set $N = \{1, 2, 3, ...\}$.
- The **complement of a set** A is the set of elements in the universe set U that are not in the set A. We will write $\bar{A}$ for the complement of the set A. Example: What is the complement of people in the class? What is the complement of the set $\{a, b, c\}$? What is the complement of a universal set? Examples 2.2 on p. 82.
- The **empty set** is the set with no elements in it. We write it as $\{\}$ or $\emptyset$.
- The set A is **subset** of B if every element of A is also an element of B. If A is a subset of B we will write $A \subset B$. Example: the set of students in the class is a subset of the people at UK which is a subset of the people currently in Kentucky.

- The **union** of the sets A and B is the set which contains all the elements of *both* A and B. The union of A and B is a set and we write it as $A \cup B$. In set builder:

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

  Example.: $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

- The **intersection** of the sets A and B is the set which contains all the elements which A and B have in common. The intersection of A and B is a set and we name it $A \cap B$. In set builder:

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

  Example.: $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$

- Two sets A and B are called **disjoint** if they have no elements in common. This is equivalent to $A \cap B = \emptyset$.

- For practice on these concepts look at the elements of the set of examples $\{2.2, 2.3, 2.4\}$ in your text.

Activity: Set Skits

1. Class is divided into groups of 5.
2. Each person in the group gets one of 5 different colored stickers. Say: red, green, blue, orange, yellow.
3. Each group gets a card with several set operations in list notation. Example: $\{red, blue\} \cup \{yellow, orange\}$.
4. Groups discuss their operations.
5. Each group presents their operations to the class by forming the result of the operation at the front of the room. Example: if the students have $\{red, blue\} \cup \{yellow, orange\}$, they should make the formation $\{red, blue, yellow, orange\}$ at the front of the room.