

Math 2250 Maple Project 7, Spring 2007. Tacoma Narrows.

NAME \_\_\_\_\_ CLASSTIME \_\_\_\_\_ VERSION A-K or L-Z

Circle the version - see problem L7.1. There are three (3) problems in this project. Please answer the questions A, B, C, ... associated with each problem. The original worksheet "2250mapleL7-S2007.mws" is a template for the solution; you must fill in the code and all comments. Sample code can be copied with the mouse. Use pencil freely to annotate the worksheet and to clarify the code and figures.

The problem headers for the Spring 2007 revision of David Eyre's project (original was year 2000).

- \_\_\_\_\_L7.1. UNDAMPED FORCED OSCILLATIONS.
- \_\_\_\_\_L7.2. PRACTICAL RESONANCE.
- \_\_\_\_\_L7.3. MCKENNA NON-HOOKES LAW CABLE MODEL.

#### L7.1. PROBLEM (UNDAMPED FORCED OSCILLATIONS )

FORCED LINEAR OSCILLATIONS. Consider the undamped forced problem

$$mx'' + kx = 9 \cos(\omega t),$$
$$x(0)=0, \quad x'(0)=0,$$

where  $m$ ,  $k$  and  $\omega$  are non-negative constants. Values  $m$ ,  $k$  are defined by the first letter of your last name:

Version A-K:  $m=1$ ,  $k=2.5$                       Version L-Z:  $m=2$ ,  $k=3.5$

- A. Choose the forcing angular frequency  $\omega$  to be 5 times larger than the natural angular frequency  $\omega_0$ ,  $\omega_0^2=k/m$ . Solve for  $x(t)$  using `dsolve()`. Plot the solution  $x(t)$  on a suitable interval in order to show the global behavior of the solution  $x(t)$ . See Figure 5.6.2, page 350. Reference: Example 1, page 350.
- B. The solution  $x(t)$  in part A is the sum of two functions, one of period  $2\pi/\omega$  and the other of period  $2\pi/\omega_0$ . Display the exact period, as calculated from the solution formula for  $x(t)$  -- see page 350, Example 1, for details about least common multiples.
- C. Suggest a value for the forcing frequency  $\omega$  so that the oscillations exhibit resonance. Show resonant behavior on a graph. Check against Figure 5.6.4, page 352.

```
# Use semicolons to see what you have done.
m:=???; k:=???;                      # Define mass and Hooke's const.
w:=5*sqrt(k/m);                      # w = 5 times w0
de:=m*diff(x(t),t,t)+k*x(t)=9*cos(w*t); # Define the differential equation
ic:=x(0)=0,D(x)(0)= 0;                # Define the initial conditions
p:=dsolve({de,ic},x(t),method=laplace); # Symbolically solve for x(t)
X:=unapply(rhs(p),t);                 # Make X(t)= the dsolve answer
a:=0: b:=???;                         # Define the plot domain
plot(X(t),t=a..b);                    # Plot the solution
>
```

```
> #L7.1-A
> #L7.1-B
> #L7.1-C
>
```

### L7.2. PROBLEM (PRACTICAL RESONANCE)

Consider the damped forced problem

$$m x'' + c x' + k x = 9 \cos(\omega t),$$

$$x(0)=0, \quad x'(0)=0.$$

Depending on the first letter of your last name, assume:

Version A-K:  $m=1, k=25$                       Version L-Z:  $m=2, k=36$

- A. Consider the damping constants  $c=2, c=1$  and  $c=1/2$ . Compute the amplitude function  $C(\omega)$  [(21), page 355] for these three equations, then plot for  $\omega=0$  to  $\omega=20$  the three amplitude graphs on a single set of axes. Compare against Figure 5.6.9 page 357 of E&P (it has one curve, yours has 3 curves).
- B. For each case  $c=2, c=1, c=1/2$ , print the values  $\omega^*, C^*$  where  $C^*=C(\omega^*)=\max \{C(\omega) : 0 \leq \omega \leq 20\}$ . The three data pairs should show that  $C^*$  becomes larger as  $c$  tends to zero. SAVE YOUR MAPLE FILE FREQUENTLY

Maple Hint: Use Maple's mouse interface on the graphic of Part C. Specifically, click on a possible maximum (horizontal tangent) in the graph to display the values  $\omega^*, C^*$  on the screen. Copy the values into a worksheet comment.

#### # EXAMPLE

```
F:=??? : m:=??? : k:=??? : unassign('c','w');
C:=unapply(F/sqrt((k-m*w*w)^2+(c*w)^2),(w,c));
plot({C(w,2),C(w,1),C(w,1/2)},w=0..15,color=black);
>
> #L7.2-A Plot C(w), three graphics on one set of axes
> #L7.2-B Table of six data values for w*, C*
>
```

### L7.3. PROBLEM (NONLINEAR MCKENNA MODELS)

There are six (6) parts L7.3A to L7.3F to complete. Mostly, this is mouse copying. Retyping the maple code by hand is not recommended.

#### NONLINEAR TORSIONAL MODEL WITH GEOMETRY INCLUDED.

Consider the nonlinear, forced, damped oscillator equation for torsional motion, with bridge geometry included,

$$x'' + 0.05 x' + 2.4 \sin(x)\cos(x) = 0.06 \cos(12 t/10),$$

$$x(0) = x_0, \quad x'(0) = v_0$$

and its corresponding linearized equation

$$x'' + 0.05 x' + 2.4 x = 0.06 \cos(12 t/10),$$

$$x(0) = x_0, \quad x'(0) = v_0.$$

The spring-mass system parameters are  $m=1$ ,  $c = 0.05$ ,  $k = 2.4$ ,  $w = 1.2$  ,  
 $F = 0.06$ . Maple code used to solve and plot the solutions appears below.

```
# Use "copy as maple text" for maple 6+.
x0:=0: a:=200: b:=300: # For part A. Change it for part B!
v0:=0: m:=1: F := 0.06: w := 1.2: m:=1: c:= 0.05: k:= 2.4:
with(DEtools): opts:=stepsize=0.1:
deLinear:= m*diff(x(t),t,t) + c*diff(x(t),t) + k*x(t) = F*cos(w*t):
ICLinear:=[[x(0)=x0,D(x)(0)=v0]]:
DEplot(deLinear,x(t),t=a..b,ICLinear,opts,title='Linear');
deNonLinear:= m*diff(x(t),t,t) + c*diff(x(t),t) +
              k*sin(x(t))*cos(x(t)) = F*cos(w*t):
ICnonlinear:=[[x(0)=x0,D(x)(0)=v0]]:
DEplot(deNonLinear,x(t),t=a..b,ICnonlinear,opts,title='NonLinear');
```

- A. Let  $x_0=0$ ,  $v_0=0$ . Plot the solutions of the linear and nonlinear equations from  $t=200$  to  $t=300$ . These plots represent the steady state solutions of the two equations.
- B. Let  $x_0=1.2$ ,  $v_0=0$ . Plot the solutions of the linear and nonlinear equations from  $t=220$  to  $t=320$ . These plots represent the steady state solutions of the two equation, with new starting value  $x_0=1.2$ . [You must modify line 1 of the maple code!]

The two linear plots in A and B have to be identical to the plot of  $x_{ss}(t)$ . The reason is the superposition formula (see E&P)  $x(t)=x_h(t)+x_{ss}(t)$ , even though the homogeneous solution  $x_h(t)$  is different for the two plots. This is because  $x_h(t)$  has limit zero at  $t=\infty$ .

- C. Determine the ratio of the apparent amplitudes (a number  $> 1$ ) for the nonlinear plots in A and B. Do "large sustained oscillations" appear in the plot of the nonlinear steady-state?

```
>
> #L7.3-A
> #L7.3-B
> #L7.3-C
>
```

#### MCKENNA'S NON-HOOKE'S LAW CABLE MODEL FOR THE TACOMA NARROWS BRIDGE

The model of McKenna studies the bridge with a nonlinear, forced, damped oscillator equation for torsional motion that accounts for the non-Hooke's law cables coupled to the equations for vertical motion. The equations in this case couple the torsional motion with the vertical motion. The equations are:

$$\begin{aligned} x'' + c x' - k G(x,y) &= F \sin wt, & x(0) &= x_0, & x'(0) &= x_1, \\ y'' + c y' + (k/3) H(x,y) &= g, & y(0) &= y_0, & y'(0) &= y_1, \end{aligned}$$

where  $x(t)$  is the torsional motion and  $y(t)$  is the vertical motion. The functions  $G(x,y)$  and  $H(x,y)$  are the models of the force generated by the cable when it is contracted and stretched. Below is sample code for writing the differential equations and for plotting the solutions. It is ready to copy with the mouse.