

- If a function $f(x)$ is differentiable @ point a , then $f(x)$ is continuous @ a
- If $f(x)$ is not continuous @ point a , then $f(x)$ is not differentiable @ a

- $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$
- $\lim_{x \rightarrow 0} \frac{\tan(x)}{x} = 1$

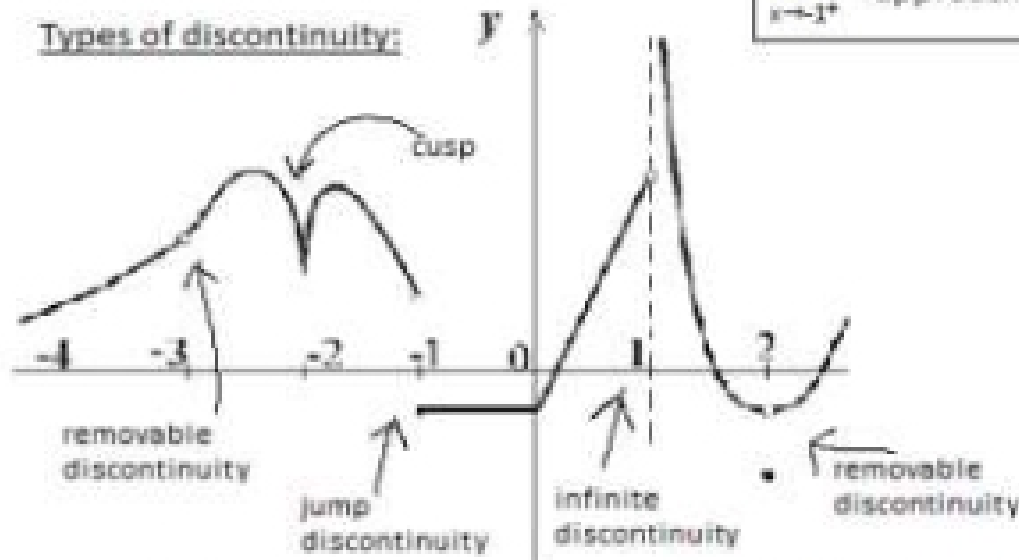
Derivative:

- aka instantaneous rate of change
- aka slope of tangent line

$\lim_{x \rightarrow 1^-}$ = approach from left

$\lim_{x \rightarrow 1^+}$ = approach from right

Types of discontinuity:



The limit of $f(x)$ does NOT exist at -1 and 1
 The function $f(x)$ is NOT continuous at -3, -1, 1, & 2
 The function $f(x)$ is NOT differentiable at -2, -1, & 1

Derivatives of Trig Functions:

- $\sin x = \cos x$
- $\tan x = \sec^2 x$
- $\sec x = \sec x \tan x$
- $\cos x = -\sin x$
- $\cotan x = -\operatorname{cosec}^2 x$
- $\operatorname{cosec} x = -\operatorname{cosec} x \cotan x$

Finding Limit of an Absolute Value:

$\lim_{x \rightarrow -1} \frac{|x+1|}{x+1}$; $|x+1| = \begin{cases} x+1 & \text{if } x \geq -1 \\ -(x+1) & \text{if } x < -1 \end{cases}$

$\lim_{x \rightarrow -1^-} \frac{|x+1|}{x+1} = \lim_{x \rightarrow -1^-} \frac{-(x+1)}{x+1}$

&

$\lim_{x \rightarrow -1^+} \frac{|x+1|}{x+1} = \lim_{x \rightarrow -1^+} \frac{x+1}{x+1}$

Finding differentiability of piecewise functions:

• $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ x^2 - 2 & \text{if } x \geq 1 \end{cases}$

$f(x) = 2x$	$f(x) = x^2 - 2$
$f'(x) = 2$	$f'(x) = 2x$
$f'(1) = 2$	$f'(1) = 2$

$2 = 2$, therefore it is differentiable at $x = 1$

Limit Laws:

- $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$

Finding continuity of piecewise functions:

• $f(x) = \begin{cases} x-2 & \text{if } x < 0 \\ x^2-2 & \text{if } x \geq 0 \end{cases}$

$f(x) = x-2$	$f(x) = x^2-2$
$f(0) = 0-2 = -2$	$f(0) = 0^2-2 = -2$

$-2 = -2$, therefore it is continuous at $x = 0$

Differentiation Rules:

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x) = 1$
- $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$
- $\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- $f(g(x))' = f'(g(x)) \cdot g'(x) \cdot x'$
- $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
- $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$
- $\frac{d}{dx}[cf(x)] = c \frac{d}{dx}f(x)$