EE513
Audio Signals and Systems

Complex Oscillator

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Oscillator Design

Marginally Stable approach: Design a system by placing poles so that a marginally stable system results, which oscillates with a fundamental frequency of f_0 when excited by a unit impulse.

Show that TF and difference equation of oscillator system are given by:

$$\hat{H}(z) = \frac{K}{1 - 2\cos(\alpha)z^{-1} + z^{-2}}$$

$$y[n] = 2\cos(\alpha)y[n-1] - y[n-2] + Kx[n]$$

where K scales the input and relates to the amplitude of oscillations, and α relates to the frequency of oscillation f_o and sampling frequency f_s by:

$$\alpha = \frac{2\pi f_o}{f_s}$$

Oscillator Design

Trig-Identity Approach: Design a system by selecting values of A and B in the trig-identity below so that $y[n]$ can substitute out the $\cos(nT\omega_0)$ function (T is sampling period) and result in a second order autoregressive difference equation.

$$\cos(A)\cos(B) = \frac{\cos(A+B)+\cos(A-B)}{2}$$

Show that difference equation of oscillator system is given by:

$$y[n] = 2\cos(T\omega_0)y[n-1] - y[n-2]$$

Oscillator is initiated with non-zero initial conditions. For n=0, let

$$y[-1] = K\cos(-T\omega_0), \quad y[-2] = K\cos(-2T\omega_0)$$

$$\omega_0 = 2\pi f_0, \quad T = \frac{1}{f_s}$$