6.003: Signals and Systems

CT Feedback and Control

October 27, 2009

Mid-term Examination #2

Tomorrow, October 28, 7:30-9:30pm, Walker Memorial.

No recitations tomorrow.

Coverage: cumulative with more emphasis on recent material
lectures 1–12
homeworks 1–7

Homework 7 includes practice problems for mid-term 2.
It will not collected or graded. Solutions are posted.

Closed book: 2 pages of notes ($8\frac{1}{2} \times 11$ inches; front and back).

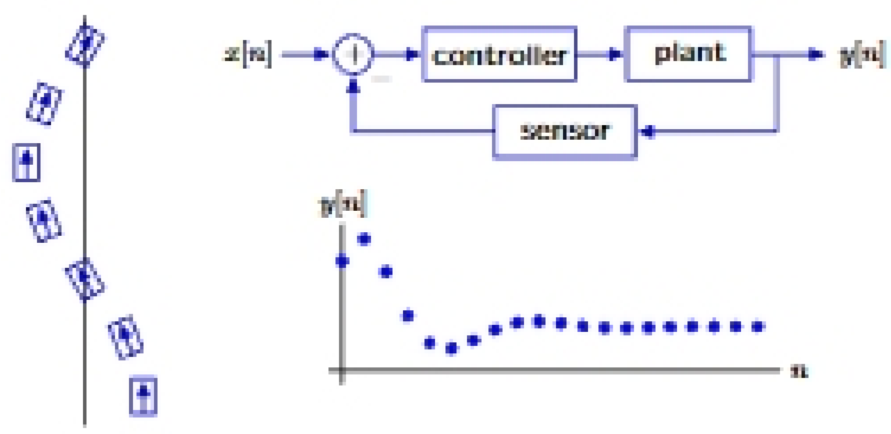
Designed as 1-hour exam; two hours to complete.

Feedback and Control

Feedback is pervasive in natural and artificial systems.

We have previously investigated feedback in DT systems.

Example: robotic steering



Feedback and Control

Today: investigate continuous-time feedback and control.

Examples:

- improve performance of an op amp circuit.
- control position of a motor.
- reduce sensitivity to unwanted parameter variation.
- reduce distortions.
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum

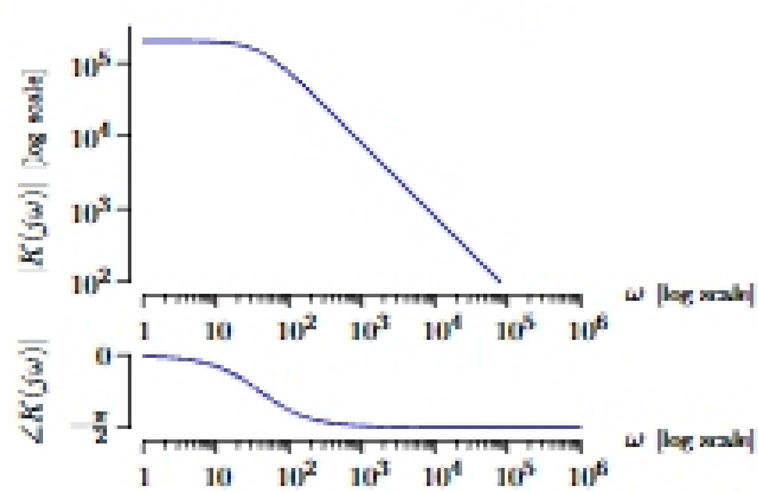
Op-Amp

Much of the utility of op-amps depends on their having high gain.

Ideal op-amp: $\lim K \to \infty$

Op-Amp

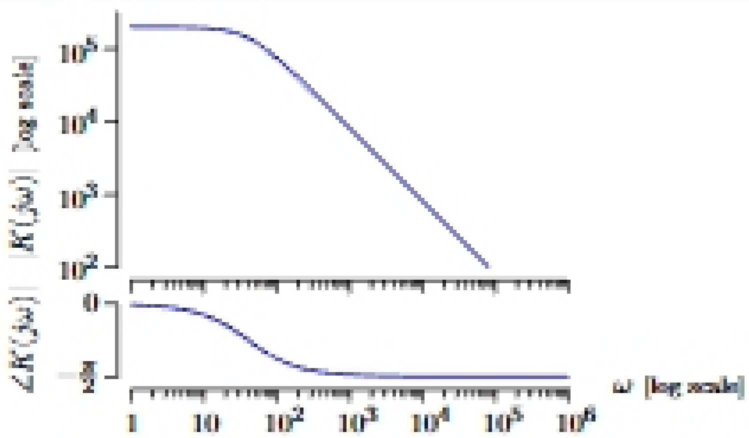
The gain of an op-amp depends on frequency.



Frequency dependence of LM741 op-amp.

Check Yourself

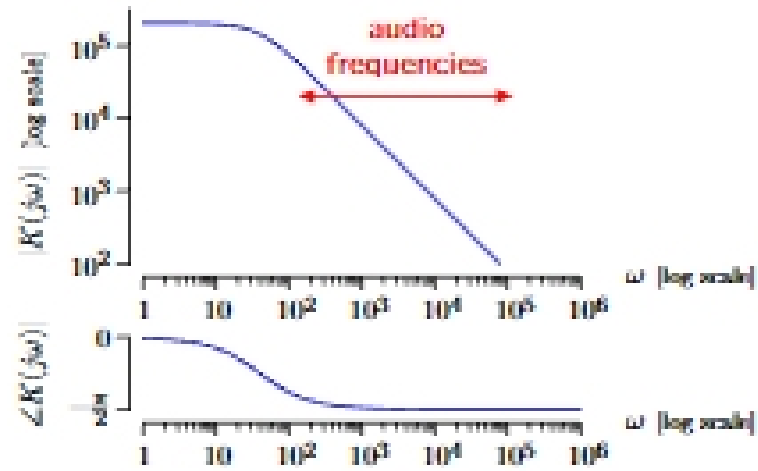
What system function has this frequency response?



1. $\frac{\alpha K_0}{s+\alpha}$ 2. $\frac{K_0 s}{s+\alpha}$ 3. $\frac{K_0(s+\alpha)}{s}$ 4. $\frac{K_0(s+\alpha)}{\alpha}$ 5. none

Op-Amp

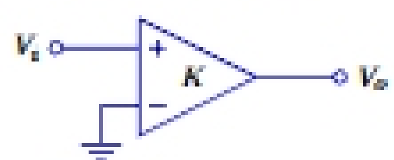
Low-gain at high frequencies limits applications.



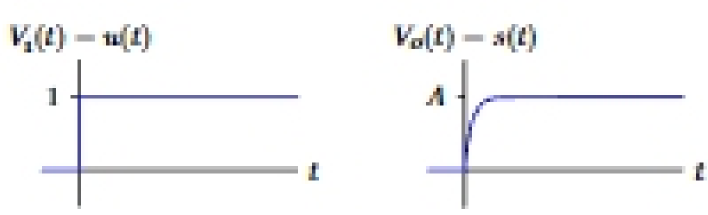
Unacceptable frequency response for an audio amplifier.

Op-Amp

An ideal op-amp has fast time response.

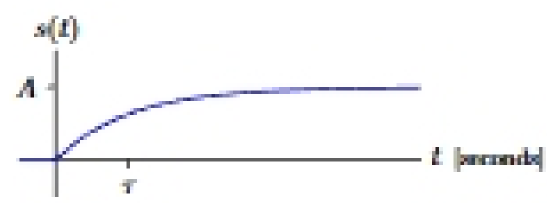


Step response:



Check Yourself

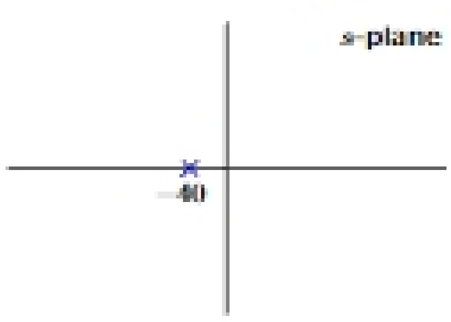
Determine the step response of an LM741.



1. $A = 2 \times 10^5$; $\tau = 40\,s$
2. $A = 2 \times 10^5$; $\tau = \frac{1}{40}\,s$
3. $A = 80 \times 10^5$; $\tau = 40\,s$
4. $A = 80 \times 10^5$; $\tau = \frac{1}{40}\,s$
5. none of the above

Op-Amp

The op-amp's poor performance parameters result from a dominant pole at a relatively low frequency.



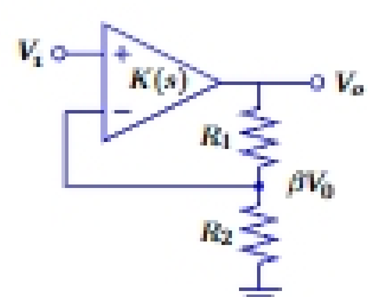
$$\alpha = 40\,\text{rad/s} = \frac{40\,\text{rad/s}}{2\pi\,\text{rad/cycle}} \approx 6.4\,\text{Hz}$$

Op-amps are designed to have a dominant pole:
→ easy to customize their behavior using feedback!

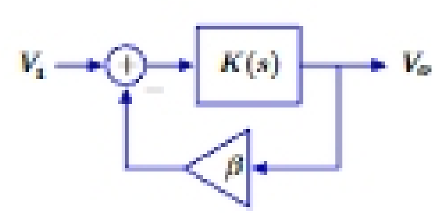
Improving Performance

Using feedback to improve performance parameters.

circuit



LTI model

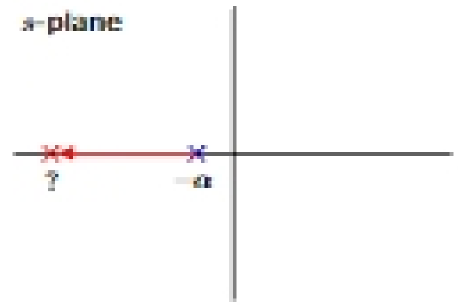


$$V_- = \beta V_o = \left(\frac{R_2}{R_1 + R_2}\right) V_o$$

$$\frac{V_o}{V_i} = \frac{K(s)}{1 + \beta K(s)} = \frac{\frac{\alpha K_0}{s+\alpha}}{1 + \beta \frac{\alpha K_0}{s+\alpha}} = \frac{\alpha K_0}{s + \alpha + \alpha \beta K_0}$$

Check Yourself

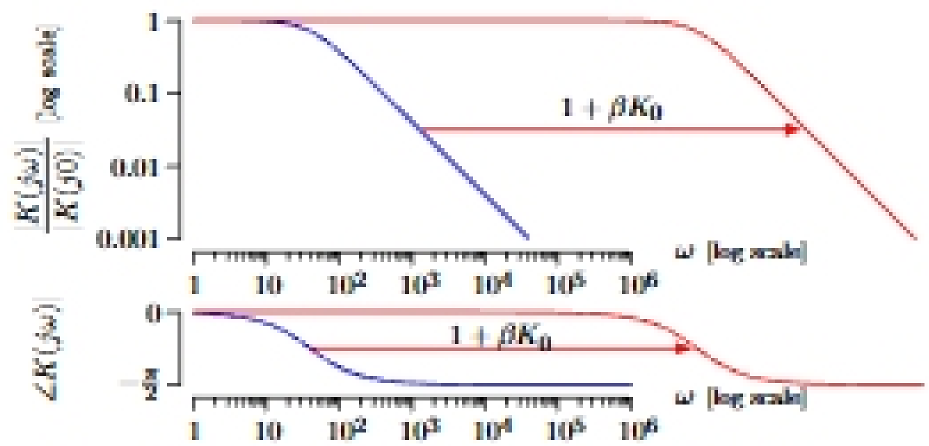
What is the most negative value of the closed-loop pole that can be achieved with feedback?



1. $-\alpha(1+\beta)$
2. $-\alpha(1+\beta K_0)$
3. $-\alpha(1+K_0)$
4. $-\infty$
5. none of the above

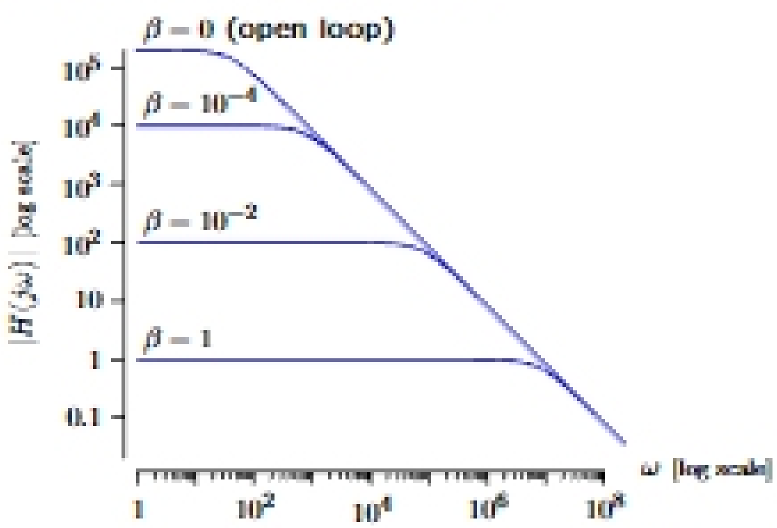
Improving Performance

Feedback extends frequency response by a factor of $1+\beta K_0$ ($K_0 = 2\times 10^5$).



Improving Performance

Feedback produces higher bandwidths by **reducing** the gain at low frequencies. It trades gain for bandwidth.

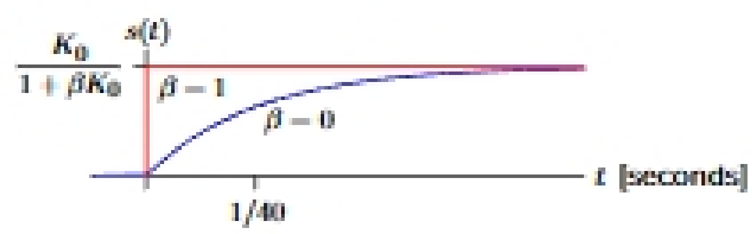


Improving Performance

Feedback makes the time response faster by a factor of $1+\beta K_0$ ($K_0 = 2\times 10^5$).

Step response

$$s(t) = \frac{K_0}{1+\beta K_0}(1-e^{-\alpha(1+\beta K_0)t})u(t)$$

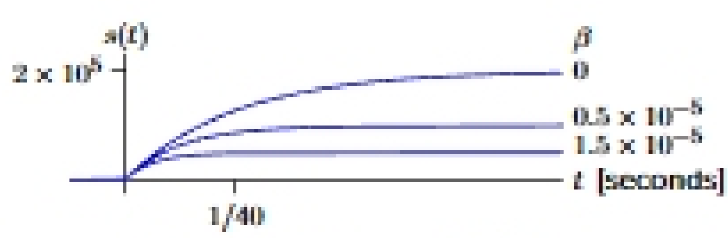


Improving Performance

Feedback produces faster responses by **reducing** the final value of the step response. It trades gain for speed.

Step response

$$s(t) = \frac{K_0}{1+\beta K_0}(1-e^{-\alpha(1+\beta K_0)t})u(t)$$



The maximum rate of voltage change $\left.\frac{ds(t)}{dt}\right|_{t=0+}$ is not increased.

Improving Performance

Feedback improves performance parameters of op-amp circuits.

- can extend frequency response
- can increase speed

Performance enhancements are achieved through a reduction of gain.