EE 422
Fall 2007

Laplace Transform (Chap. 15 + 16)

The FT can be considered a special case the Laplace Transform

$$\mathcal{L}\{x(t)\} = \hat{X}(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

To get FT let $s = j2\pi f$

in general $s = \sigma + j\omega$ (complex frequency)

$$\hat{X}(s) = \int_{-\infty}^{\infty} x(t) e^{-(\sigma + j\omega)t} dt$$

$$= \int_{-\infty}^{\infty} \left(x(t) e^{-t\sigma}\right) e^{-j\omega t} dt$$

integral may not exist for some $x(t)e^{-t\sigma}$ combinations as well as ranges of σ

Region of Convergence
<u>ROC</u>

Consider a system with impulse response $h(t)$ and input signal $Ae^{st} = x(t)$

Note by Eulers $Ae^{st} = Ae^{\sigma t} e^{j\omega t}$

$$= Ae^{\sigma t}(\cos(\omega t) + j\sin(\omega t))$$

Find the output.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) A e^{(t-\tau)s}\, d\tau$$

$$= A e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-\tau s}\, d\tau}_{\text{LT of } h(\tau)}$$

also known as the Transfer functions

Example find LT of

$t\,u(t) \Rightarrow \text{ramp}(t)$

$$\hat{X}(s) = \int_{-\infty}^{\infty} t e^{-st} u(t)\, dt = \int_{0}^{\infty} t e^{-st}\, dt$$

$\int f(x) g'(x)\, dx = f(x) g(x) - \int g(x) f'(x)\, dx$ use Table or integrate by parts

$t = u \quad dv = e^{-st}$

$du = 1 \quad v = -\frac{1}{s} e^{-st}$

$$\left. \frac{t}{s} e^{-st} \right|_0^{\infty} + \frac{1}{s}\int_0^{\infty} e^{-st}\, dt = 0 + \frac{1}{s} \left. \frac{-1}{s} e^{-st} \right|_0^{\infty} \Rightarrow \frac{1}{s^2}$$

$$\int_0^{\infty} e^{-\sigma t} e^{-j\omega t}\, dt$$

exists only for $\sigma > 0$

ROC $Re(s) > 0$

Ex1 Do problem 2a + 2b

To obtain FT from Laplace
let $s = j\omega$ (ie. $\sigma = 0$)

Since Laplace will be applied primarily to causal impulse responses and signals with positive time axes, the unilateral Laplace transform will be used

$$\hat{X}(s) = \int_0^\infty x(t) e^{-st} dt$$

Note the 2-sided or bilateral Laplace becomes the unilateral Laplace transform if the unit step function is part of the time function.

Properties of Laplace Transforms
see pages 585 to 591.

Convolution ↔ Multiplication
show $g(t) * h(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \hat{G}(s)\hat{H}(s)$