

Section 5.1 – Simple Interest, Future Value, Present Value, and Effective Rate

Simple Interest is interest that is compounded on the original principal only.

P = principal

r = interest rate per year (% to decimal)

t = time (in years)

Accumulated Amount is the sum of the principal and interest after t years.

Formula: $A = P + I$
 $A = P + Prt$
 $A = P(1 + rt)$

Example 1: Find the simple interest on a \$1000 investment made for 3 years at an interest rate of 5% per year. What is the accumulated amount?

$$A = P(1 + rt)$$

$$A = 1000(1 + .05(3))$$

$$A = \$1,150.00$$

Example 2: Find the simple interest rate at which \$1,000 will grow to \$1,050.

In nine months $A = P(1 + rt)$ $.05 = \frac{3}{4}r$

$$A = 1,050 \quad r = ?$$

$$P = 1,000 \quad t = \frac{9}{12} = \frac{3}{4}$$

$$1050 = 1000(1 + r(\frac{3}{4}))$$

$$1.05 = 1 + r(\frac{3}{4}) \quad r = .067$$

6.7%

Compounded Interest is earned interest that is periodically added to the principal and there after earns interest at the same rate.

$$A = P(1 + i)^n \quad i = \frac{r}{m} \quad n = m \cdot t$$

A stands for the future value or the accumulated amount at the end of n conversion periods. A conversion period refers to the interval of time between successive interest calculations.

P stands for the present value or principal.

r stands for the interest rate per year.

m stands for the number of compounding periods per year.

t stands for time (years).

FV of Compound

Example 3: Find the **accumulated amount** after 5 years if \$1700 is invested at 6.25% per year compounded

$$A = P(1 + i)^n$$

a. quarterly. $\rightarrow m$

$$P = 1700$$

$$i = \frac{r}{m} = \frac{.0625}{4}$$

$$r = .0625$$

$$m = 4$$

$$n = m \cdot t = 4(5) = 20$$

$$A = 1700 \left(1 + \frac{.0625}{4}\right)^{20}$$

$$A = \$2318.02$$

b. semiannually. $\rightarrow m$

$$i = \frac{r}{m} = \frac{.0625}{2}$$

$$n = m \cdot t = 2(5) = 10$$

$$A = 1700 \left(1 + \frac{.0625}{2}\right)^{10}$$

$$A = \$2312.54$$

Present Value

Recall $A = P(1 + i)^n$ and $P =$ Present Value

Why would we want to find P ?

Well in certain instances an investor may wish to determine how much money he should invest now, at a fixed rate of interest, so that he/she will realize a certain sum of money at some future date.

So, solving the Future Value formula for P we obtain the Present Value with compound interest formula.

$$A = P \left(1 + \frac{r}{m}\right)^n \Rightarrow P = \frac{A}{\left(1 + \frac{r}{m}\right)^n} \Rightarrow P = A \left(1 + \frac{r}{m}\right)^{-n}$$

Example 4: Kim and Ken find that they will need \$15,500 to build an addition to their home in 4 years. **How much should they invest now** at 3.25% per year compounded **quarterly** to have the desired funds in **4 years**?

$$P = A(1 + i)^{-n}$$

$$A = 15,500$$

$$r = .0325$$

$$m = 4$$

$$n = m \cdot t = 4(4) = 16$$

$$P = 15,500 \left(1 + \frac{.0325}{4}\right)^{-16}$$

$$P = \$13,617.63$$

Example 5: A newborn child receives a \$5000 gift towards a college education from her grandparents. **How much will the \$5000 be worth in 17 years if it is invested at 9% per year compounded quarterly?**

$$A = P(1 + i)^n$$

$$P = 5,000$$

$$r = .09$$

$$m = 4$$

$$n = m \cdot t = 4(17) = 68$$

$$A = 5,000 \left(1 + \frac{.09}{4}\right)^{68}$$

$$A = \$22,702.60$$

Example 6: An Individual Retirement Account (IRA) has \$20,000 in it and the owner decides not to add any more money to the account other than interest earned at 8% per year **compounded monthly**. **How much will be in the account 35 years** from now when the owner reaches retirement age?

$$A = P(1 + i)^n$$

$$P = 20,000$$

$$r = .08$$

$$m = 12$$

$$n = m \cdot t = 12(35) = 420$$

$$A = 20,000 \left(1 + \frac{.08}{12}\right)^{420}$$

$$A = \$325,851.00$$

Example 7: Kim invested a sum of money 4 years ago in a savings account that has since paid interest at the rate of 6.5% per year compounded monthly. Her investment is now worth \$19,440.31. **How much did she originally invest?**

$$P = A(1 + i)^{-n}$$

$$A = 19,440.31$$

$$r = .065$$

$$m = 12$$

$$n = m \cdot t = 12(4) = 48$$

$$P = 19,440.31 \left(1 + \frac{.065}{12}\right)^{-48}$$

$$P = \$15,000.00$$