Eigen Decomposition and Singular Value Decomposition

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Introduction

- Eigenvalue decomposition
  - Spectral decomposition theorem
- Physical interpretation of eigenvalue/eigenvectors
- Singular Value Decomposition
- Importance of SVD
  - Matrix inversion
  - Solution to linear system of equations
  - Solution to a homogeneous system of equations
- SVD application

What are eigenvalues?

- Given a matrix, **A**, **x** is the eigenvector and λ is the corresponding eigenvalue if $\mathbf{Ax} = \lambda\mathbf{x}$
  - A must be square the determinant of $\mathbf{A} - \lambda\,\mathbf{I}$ must be equal to zero

$$\mathbf{Ax} - \lambda\mathbf{x} = 0 \rightarrow \mathbf{x}(\mathbf{A} - \lambda\mathbf{I}) = 0$$

    - Trivial solution is if $\mathbf{x} = 0$
    - The non trivial solution occurs when $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$
- Are eigenvectors are unique?
  - If **x** is an eigenvector, then $\beta\mathbf{x}$ is also an eigenvector and $\beta\lambda$ is an eigenvalue

$$\mathbf{A}(\beta\mathbf{x}) = \beta(\mathbf{Ax}) = \beta(\lambda\mathbf{x}) = \lambda(\beta\mathbf{x})$$