

From the questions below, please solve 5 problems only.

The homework is worth 10 points. Each question is worth 1 points.

Show all of your work and put a box around your final answer.

Number each attempted question clearly.

Write legibly (that is, suitably large and suitably dark); if the grader can't read your answer, it's consider uncompleted.

Question 1 Calculate the derivative indicated using the given function.

(a) $f''(1)$, $f(t) = \frac{t}{1+t}$

(b) $\frac{d^3x}{dt^3}$, $x(t) = t^{-3/4}$

Question 2 Find the derivative of each function.

(a) $f(\theta) = \sec \theta + 3\theta$

(c) $g(y) = \frac{\cos y}{y^2 \sin y + 3e^y}$

(b) $f(z) = \frac{1}{4}ze^z - \frac{1}{16}e^z$

(d) $h(x) = \frac{\tan x}{x^3 + e^x}$

Question 3 Suppose that the function $f(x)$ and $f'(x)$ have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	1/5

Find the first derivatives of the following combination at the given value of x .

(a) $\frac{f(x)}{x + \cos(x)}$, $x = 0$

(b) $e^x(f(x))^2$, $x = 1$

Question 4 Use the quotient rule to derive the formula for $\frac{d}{dx} \csc x$ and for $\frac{d}{dx} \cot x$.

Question 5 Some values of a differentiable, invertible function f and its derivative are given below.

x	0	2	4	6	8
$f(x)$	15	13	10	6	2
$f'(x)$	-1	-2	-2.5	-3	-5

Calculate the following.

(a) $g'(2)$, where $g(x) = f(x)e^{2x}$.

(b) $h'(6)$, where $h(x) = xe^{f(x)}$.

Question 6 Use the table below to evaluate the derivatives below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
1	2	3	4	5	-2	-1

(a) Let $F(x) = f(x)g(x)$ and find $F'(1)$.

(b) Let $G(x) = \frac{g(x)}{f(x)}$ and find $G'(1)$.

(c) Let $H(x) = f(x)g(x)h(x)$ and find $H'(1)$.

(d) Let $J(x) = \frac{f(x)g(x)}{h(x)}$ and find $J'(1)$.

Question 7 A mass is hanging from the ceiling on a spring and oscillating (forever, and without friction). At its highest point, the mass is 12 cm from the floor, and at its lowest point, the mass is 6 cm from the floor. At $t = 0$, the mass is at its equilibrium and moving downwards, and a single period takes 0.5 seconds. HINT: Graph your $y(t)$ to check your answers.

- (a) Find a formula for $y(t)$, the position of the mass from the floor as a function of time.
- (b) What is the position, velocity, and acceleration of the object at $t = 2$?
- (c) In its first period, during what time interval(s) is the *speed*, $|v|$, of the mass less than 1 cm/s?

Question 8 The fuel efficiency of a car $E(s)$, in miles per gallon, is a function of the speed of the car s , in miles per hour. Suppose that $E(60) = 40$ and $E'(60) = -0.7$.

- (a) What are the units of $E'(s)$?
- (b) Let $C(s)$ be the fuel consumption, in gallons per mile, of the car when it is driving s miles per hour. Find a formula for $C(s)$ in terms of $E(s)$. Calculate $C'(60)$.
- (c) Let $R(s)$, be the rate the car burns fuel, in gallons per hour, when the car is driving s miles per hour. Find a formula for $R(s)$ in terms of $E(s)$. Calculate $R'(60)$.

Question 9 In 1999, the population of Richmond-Petersburg, Virginia, metropolitan area, was 961,400 and was increasing at a at roughly 9200 people per year. The average annual income in the area was \$30,593 per capita, and this average was increasing at about \$1400 per year. Use the product rule to estimate the rate at which total personal income was rising in the area at this time. Explain the meaning of each term in the product rule.

Question 10 The revenue per month earned by the Couture clothing chain at time t is

$$R(t) = N(t)S(t),$$

where $N(t)$ is the number of stores and $S(t)$ is the average revenue per store per month. Couture releases a statement to their investors saying that currently they have 50 stores with a total revenue of \$7,500,000 and expect to increase the number of stores by about 2 stores per month and increase total revenue by about \$250,000 per month. What does this statement imply about the current rate of change of the average revenue per store per month?