3650:350 **Modeling and Simulation** **Spring 2002**

Exercises #6
due: Tuesday, March 5

Reading Assignment:

- For additional information on this week's material, please read:
  Cooper, *A Matlab Companion for Multivariable Calculus* :
  Sections 4.1-4.4 (Curves in space) pp 47-58
  Section 11.2 (Line integrals) pp 223-227

- To prepare for next week, please read
  Cooper, *A Matlab Companion for Multivariable Calculus* :
  Sections 5.5-5.6 (Partial derivatives etc. and The gradient vector etc.) pp 79-86
  Section 6.3 (The gradient vector field) pp 108-110
  Section 11.1 (Vector fields) pp 219-222
  Section 11.3 (Curl and Green's theorem) pp 227-231

1. Numerical Integration

   (a) On paper: Calculate the integral of the function

$$f(x) = x\sin(x) \tag{1}$$

   for integration boundaries a and b. Evaluate for the case $a = 0$ and $b = \pi/2$.

   (b) Matlab offers symbolic integration tools in addition to numerical ones. Check your analytic results by typing the following in the command window

```
clear
syms a b x
f=x*sin(x)
int(f,x,a,b)
int(f,x,0,pi/2)
```

   Please copy the statements and Matlab's responses into your word file.

   (c) Open the files *numint.m*, *simpvec.m*, and *fxsin.m* in the editor and familiarize yourself with their content. The program calls two Matlab functions that we have not used before. To find out how the `dot` function works, first type `help dot` in the command window. Then (still in the command window) define two vectors, for example `u=[1 2 3 0]` and `v=[1 1 1 1]`, and type `dot(u,v)` and `dot(u,u)`. Please copy these statements and Matlab's responses into your word file. The Matlab routine `quad` performs numerical integrations with an adaptive Simpson method. We discussed the technique in class, but type `help quad` to find out more about the function's usage.

(d) In *numint.m*, set the number N of evaluation points for Simpson's method to $N = 51$, run the program and record the results displayed in the command window. The result from Simpson's rule should be slightly worse than the result from the `quad` routine with a tolerance `tol=1.e-6`. Increase the number N until you reach better accuracy with Simpson's rule than with `quad` and record the value N. Now decrease the number N and see if you can get a Simpson's rule estimate for the integral that differs by more than 1% from the exact value.

(e) Now consider the integral over the function

$$g(x) = \sqrt{1 - x^2} \tag{2}$$

with integration limits $a = 0$, $b = 1$. Find the analytic solution for the integral and then adapt the m-files to perform the numerical integration. In this case, how many evaluation points N do you need to match the `quad` results with Simpson's method? How few points will give you an accuracy of 1%?

(f) Can you think of a reason why it would be more difficult to perform the integral in (e) than that in (a)? Plot the integrands $f(x)$ and $g(x)$ over the range where the integration is performed and comment.

2. Find parametric representations for the following paths in the x-y plane and write a Matlab file to plot the curves.

   (a) From the origin, a straight line of length 1 along $\hat{x}$.

   (b) A straight line from $(1, 1)$ to $(0, 1)$.

   (c) A straight line from $(-1, 1)$ to $(0, 0)$.

   (d) Starting from $(1, 1)$ along the right hand side branch of the parabola $y = -(x - 1)^2 + 1$ until $y = 0$.

3. Calculate the work done by the forces due to gravity and air drag on the projectile of Problem 3. of the previous assignment (Ballistic motion: Giordano, Exercise 3. on page 28). Assume that the firing angle is 45^o and that the mass of the cannon ball is 33 kg.

Please make sure you answer the questions in the text and please don't forget the PreClass assignment on the web.