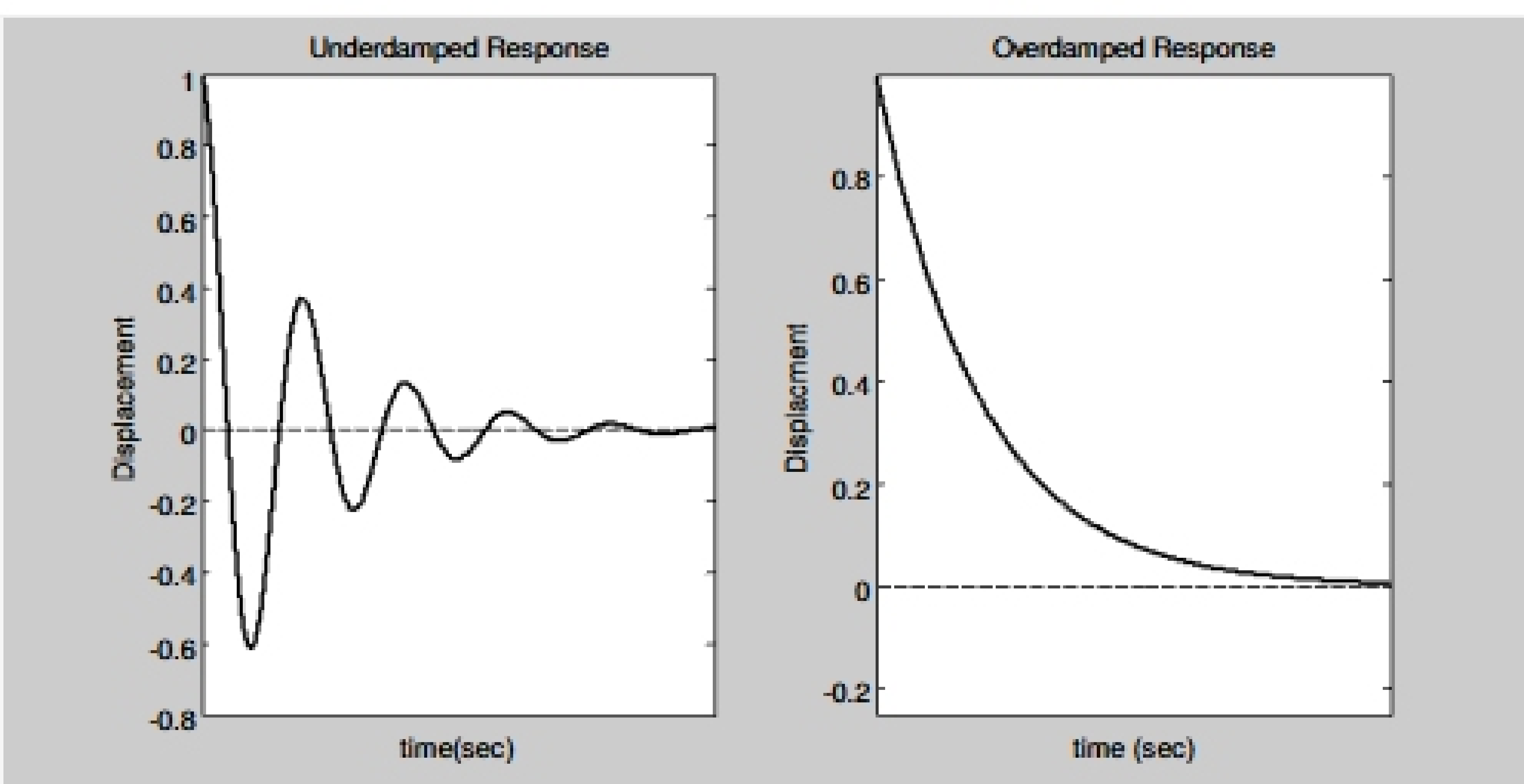
# Lab 6: Damped Sinusoidal Motion

**Damped Sinusoidal Motion**

In Lab 3, we looked at exponential functions and some examples of processes that could be modeled with exponential functions. In Lab 4, we looked at sinusoidal functions and explored the concept of simple harmonic motion. In this lab, we will combine exponentials and sinusoids and explore the concept of damped sinusoidal motion. Many physical responses can be modeled by damped sinusoidal motion. For example, the shock absorbers in a car allow the passengers to experience damped vibrations when hitting a pothole or a speed bump.

The diagram below shows a mass connected to a spring. If there were no loss of energy, compressing or stretching the spring would result in sustained oscillations (simple harmonic motion). However, in reality, there is a loss of energy in the spring and the amplitude of the oscillations will decrease over time.

Depending on the characteristics of the spring and the mass, the displacement of the mass will look like one of the two graphs shown below. The mass could oscillate around the equilibrium point with the amplitude of the oscillations becoming smaller and smaller (***underdamped response***) or the mass could simply return directly to the equilibrium point (***over-damped response***).



If the response is ***underdamped***, the displacement of the mass can be modeled mathematically as follows:

$$d(t) = d_0 e^{-\alpha t}\cos(\omega t)$$

$$\alpha = \frac{B}{2M}$$

$$\omega = \sqrt{\frac{K}{M} - \frac{B^2}{4M^2}}$$

$$B < 2\sqrt{K \cdot M}$$

**d:** displacement of the mass in meters (m)
**$d_0$:** initial displacement of the mass in meters (m)
**ω:** the frequency of oscillation (rad/s)
**M:** mass (kg)
**K:** spring constant (N/m)
**B:** damping coefficient (N·s/m)

Recall from Lab 3, the spring constant is a measure of resistance to displacement. The force that it takes to compress or stretch the spring is directly proportional to the displacement. In other words, it takes twice as much force to compress or stretch the spring by 10 cm than it would to compress of stretch the spring by 5 cm.

The damping coefficient models the energy loss in the system. The damping is proportional to the velocity of the mass. The faster the mass moves, the higher the damping force opposing that motion will be.
**Note: If $B \geq 2\sqrt{K \cdot M}$, the response will be over-damped (the mass simply returns directly back to equilibrium if damping force is large enough).**

In this lab you will write a program that will allow you to explore the effects of the spring constant, K, the damping coefficient, B, and the mass, M, on the motion of the mass.

1. Go to the recitation folder on the Blackboard metasite and download the m-file template. Save it in whatever you use as your current folder for MATLAB with a filename: Lab6_***YourLastName***. Remember, filenames follow the exact same rules as variables in MATLAB (start with a letter followed by any combination of letters, numbers, and underscores and ***no spaces allowed***!)

2. In the template, fill in your name and date where indicated. You might want to wait on the description until you have written the program.

3. Write four input statements to prompt the user for the spring constant, K, the mass, M, the damping coefficient, B, and the initial displacement, dint. Make sure you write good prompts for the user which include units. For example: `input('Enter K:  ')` is not very informative to the user. `input('Enter the spring constant, K, in N/m:   ')` is a much better prompt.

4. Write a conditional statement using the ***if* …. *else* … *end*** construction. ***If*** $B \geq 2\sqrt{K \cdot M}$, your program should inform the user that the response is over-damped. Otherwise (***else***), your program should inform the user that the response is underdamped. Use an ***fprintf*** statement to do this. Display all values using two places behind the decimal point. For example, if the response turns out to be over-damped, your program should output the following statement:

   For K = *display_value_entered_by_user*, B = *display_value_entered_by_user*, and M = *display_value_entered_by_user*, the response is over-damped.

5. Now test your program. Suppose K = 200 N/m, M = 0.5 kg, and $d_o$ = 1 m.

   Calculate the range of B that will result in an over-damped response? _________ (Nm/s)

   Calculate the range of B will result in an under-damped response? _____________ (Nm/s)

   Run your program for various values of B and see if it produces the correct result.

6. In the underdamped section of your program (***else***), add a line to compute the frequency of the oscillations, ω, in radians per second.

7. Add another line to compute the frequency of the oscillations in Hz. (Recall: ω = 2πf).

8. Add an ***fprintf*** statement that tells the user what the frequency of oscillation is in both radians per second and in Hz. Display the frequencies using ***2 places*** behind the decimal point.

9. Now test your program again using the values K = 200 N/m, M = 0.5 kg, B = 4, and $d_o$ = 1m. The frequency of oscillation should be 19.6 rad/s or 3.12 Hz.

10. The amplitude of the oscillations decay exponentially as

$$d_0 e^{-(B/(2 \cdot M))t}$$

   Find an equation for the time, T_END, at which this amplitude will be: $d_0 e^{-5}$. Your equation will depend on B and M.

   T_END = _______________ (s)