AAE 340 – Dynamics and Vibrations
Problem Set 6
Due: 10/9/13

Problem 1: A particle P (mass m) is attached to a rigid, inextensible rod that can rotate in a vertical plane. The rod is attached to a massless slider that can move along a frictionless track.

(a) Define all the appropriate quantities. Describe in words the constraints that enable the system to be modeled using only two VOI.

(b) Use $\bar{F} = m\,{}^{i}\bar{A}^{P}$ and determine one EOM. [Hint: include S in your FBD.]

$$\left\{ Ans: \quad m\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta = 0 \right\}$$

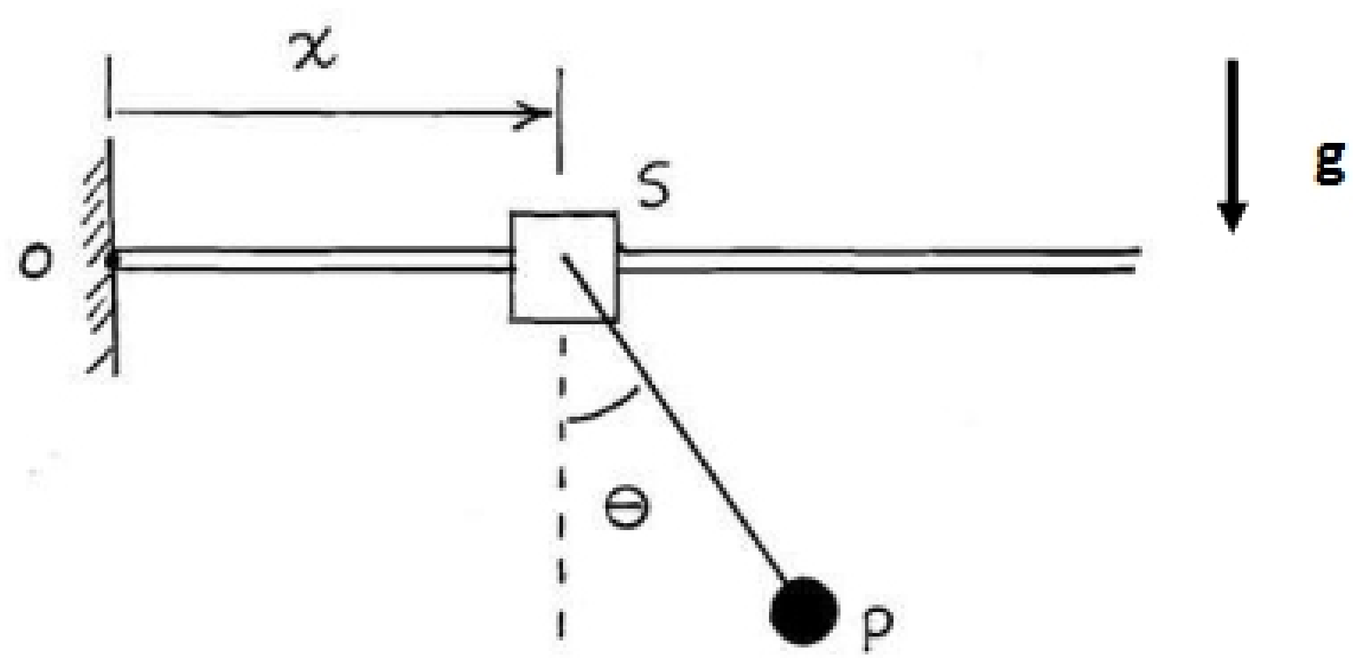
(b) Use $\bar{M}^{O} = \dfrac{{}^{i}d\,{}^{i}\bar{H}^{O}}{dt}$. Can you use it to obtain the second equation of motion?

$$\left\{ Ans: \quad \ddot{\theta} + \frac{g}{L}\sin\theta + \frac{\ddot{x}}{L}\cos\theta = 0 \right\}$$

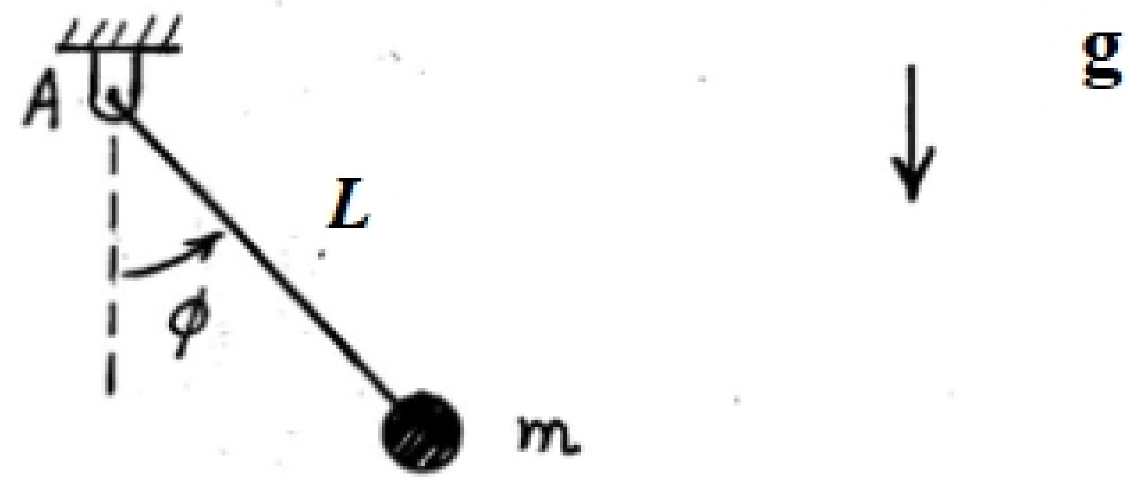
(c) A component of linear momentum is constant. Justify this statement.
Write an expression for the constant component of linear momentum. Differentiate it and identify this constant with one of the EOMs.

Is energy or angular momentum constant? Justify your answers.

(d) The system parameters are $m = 2$ kg and $L = 6$ meters. Let the initial conditions be given as $\theta(0) = 60^\circ$, $\dot{\theta}(0) = 0$, $x(0) = 1$ met, $\dot{x}(0) = 0$. Determine the rates $\dot{\theta}, \dot{x}$ when $\theta(0) = 0^\circ$.



Problem 2: A simple pendulum of length L and mass m appears below.
Let $m = 2$ kg and $L = 6$ meters.



(a) Assume small oscillations and derive the EOM.
At the initial time, $\phi(0) = 150^\circ$, $\dot{\phi}(0) = 0$. Determine the complete solution to the differential equation $\phi(t)$.
What is the natural frequency and the period?

(b) Derive the true EOM, i.e., do NOT assume small oscillations.
$$\left[Ans: \ddot{\phi} + \frac{g}{L}\sin\phi = 0 \right]$$

(c) Prove that energy is a constant of the motion.

(d) Initially, $\phi(0) = 150^\circ$, $\dot{\phi}(0) = 0$. Develop a Matlab M-file and numerically integrate the differential equation; integrate for 3 cycles. The output quantities should include t, ϕ (in degrees), $\dot{\phi}$ (in rad/s), energy.
Produce the following plots:
   (i) ϕ as a function of t
   (ii) $\dot{\phi}$ as a function of t
   (iii) $\dot{\phi}$ as a function of ϕ ← phase plot
   (iv) $\Delta E = E - E_0$ as a function of t
Each plot should have two curves — the analytical solution from (a); the numerical solution from (b).
[To evaluate a constant using numerical integration, always plot $\Delta E = E - E_0$, that is, the value of energy at time t minus the value of energy at the initial time.]

(e) From the plots, estimate the period for the numerical solution from (b). How does it compare with the period computed in (a)? Is the actual period longer or shorter that the linearized estimate? Is "small oscillations" a good assumption in this problem? Why or why not? Are there any differences besides period?

(f) Check the curves in phase space. Do both solutions appear to be periodic in the plot? How are they different? What conclusion can you make about $\dot{\phi}_{max}$ in each case? Which one is correct?

Problem 3: A circular hoop H of radius R can rotate about a horizontal shaft at the constant rate $\dot{\beta}$ rad/s. (The shaft is fixed in the inertial frame.) A bead B of mass m can move at the variable rate $\dot{\gamma}$ inside the frictionless hoop.

(a) Define all the appropriate quantities. What is the minimum number of sets of unit vectors that are required?

(b) How many EOM are required to completely describe the motion? Justify your answer.

(c) Use the force equation and derive the resulting differential equations. Which (if any) are equations of motion?

(d) Use a moment equation (about an inertially fixed point OR point Q) and derive the differential equations. Which (if any) are EOM?
Are the equations the same as in part (c)?

(e) Derive an expression for kinetic energy; potential energy due to gravity.
Do you think energy is constant? Why or why not?