Exam 1 solutions

1. a $\dfrac{1}{\frac{1}{6}x^3y} \cdot \dfrac{1}{6^2x^{10}y^{-4}z^{-2}}$

$= \dfrac{6}{x^3y} \dfrac{y^4z^2}{6^2x^{10}}$

$= \boxed{\dfrac{y^3z^2}{6x^{13}}}$

b. $64^{-1/3} = \dfrac{1}{\sqrt[3]{64}} = \dfrac{1}{4}$

$\sqrt{63} + \sqrt{28}$

$= 3\sqrt{7} + 2\sqrt{7}$

$= 5\sqrt{7}$

$\Rightarrow \dfrac{\frac{1}{4}}{(5(\sqrt{7}))^2} = \dfrac{1}{4} \cdot \dfrac{1}{25 \cdot 7} = \dfrac{1}{700}$

2. $3(x^2 - 4x + 4 - 4) + 7$

$\left(\frac{-4}{2}\right)^2 = (-2)^2$

$= 3(x^2 - 4x + 4) - 12 + 7$

$= \boxed{3(x-2)^2 - 5}$

3. $2|2x+3| + 4 > 6$

$-4 \quad -4$

$\dfrac{2}{2}|2x+3| > \dfrac{2}{2}$

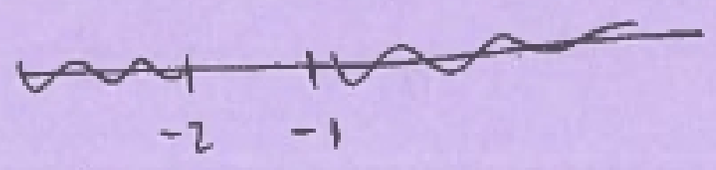
$|2x+3| > 1$

$2x+3 < -1$ OR $2x+3 > 1$

$-3 \quad -3 \qquad -3 \quad -3$

$2x < -4 \qquad 2x > -2$

$x < -2 \qquad x > -1$



$(-\infty, -2) \cup (-1, \infty)$

4. $2 - x = \sqrt{2x+31}$

$(2-x)^2 = 2x+31$ ☆

$4 - 4x + x^2 = 2x + 31$

$-31 \quad -2x \qquad -2x - 31$

$x^2 - 6x - 27 = 0$

$(x+3)(x-9) = 0$

$x = -3$

$x = 9$

Check: $5 = \sqrt{-6+31}$ ✓

$-7 = \sqrt{18+31}$ no

$\boxed{x = -3}$

5. a. $g(-1) = -(-1)^2 - 2|1-(-1)|$

$= -1 - 2|2|$

$= -5$

$g(2) = -(2)^2 - 2|1-2|$

$= -4 - 2|-1|$

$= -6$

$2g(-1) - g(2) = 2(-5) - -6$

$= -10 + 6 = \boxed{-4}$

b. $\dfrac{f(x+1) - f(x)}{2}$

$= \dfrac{(x+1)(x+2) - x(x+1)}{2}$

$= \dfrac{(x+1)(x+2-x)}{2}$

$= \dfrac{(x+1)2}{2} = \boxed{x+1}$

6. $t + 9 = 0$ would be bad

$t = -9$ BAD

$\boxed{(-\infty, -9) \cup (-9, \infty)}$

7. $\dfrac{x+12}{x+2} - x \geq 0$

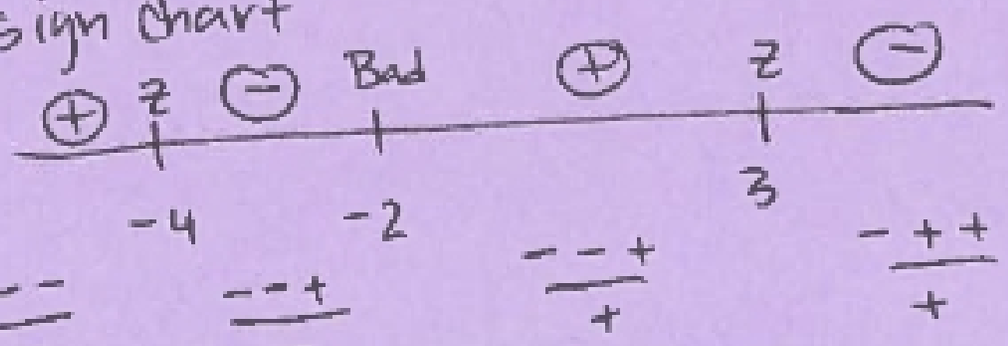
$\dfrac{x+12}{x+2} - \dfrac{x(x+2)}{x+2} \geq 0$

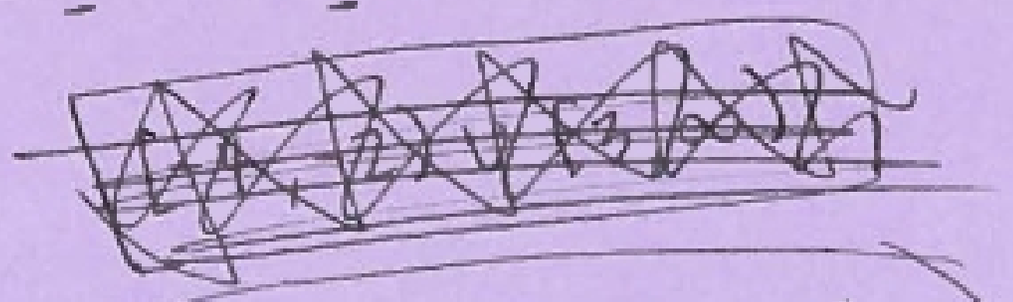
$\dfrac{x+12 - x^2 - 2x}{x+2} \geq 0$

$\dfrac{-(x^2 + x - 12)}{x+2} \geq 0$

$-\dfrac{(x-3)(x+4)}{x+2} \geq 0$

sign chart





$\boxed{(-\infty, -4] \cup (-2, 3]}$

8. $$\frac{g(-4)-g(-1)}{-4--1}$$

$$= \frac{(-4)^2+3(-4)-((-1)^2+3(-1))}{-3}$$

$$= \frac{16-12-(-2)}{-3}$$

$$= \frac{6}{-3}$$

-2

9. $x, x+2, x+4$

$2x^2+(x+2)^2 = 2(x+4)^2-39$

$2x^2+x^2+4x+4 = 2x^2+16x+32-39$

$-2x^2 \quad -16x+7 \quad -2x^2-16x+7$

$x^2-12x+11=0$

$(x-11)(x-1)=0$

$x=11, x=1$

1, 3, 5 and 11, 13, 15

10 f odd so $f(-x)=-f(x)$

then $g(-x)=2f(-x)-\sqrt[3]{-x}$

$=-2f(x)--\sqrt[3]{x}$

$=-(2f(x)-\sqrt[3]{x})$

$=-g(x)$

$\sqrt[3]{x}$ is odd

So g is odd

11. $u=x^2+x$

$u^2-8u+12=0$

$(u-6)(u-2)=0$

$u=6 \quad u=2$

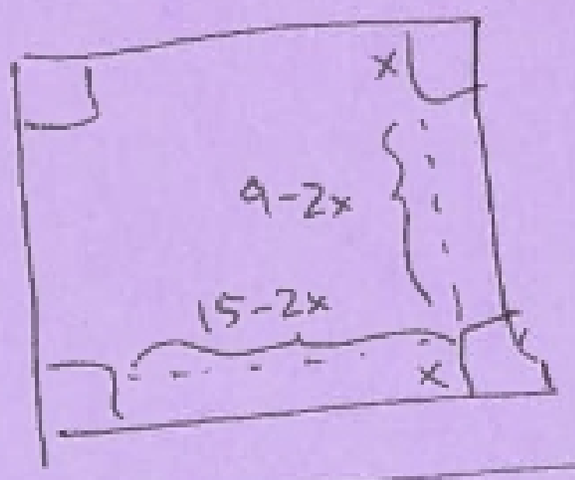
$x^2+x=6 \quad x^2+x=2$

$x^2+x-6=0 \quad x^2+x-2=0$

$(x-2)(x+3) \quad (x+2)(x-1)$

$x=2,-3 \quad x=-2,1$

Bonus



$V(x)=x(9-2x)(15-2x)$