

**AAE 340 – Dynamics and Vibrations**

**Exam I**

*Solution*

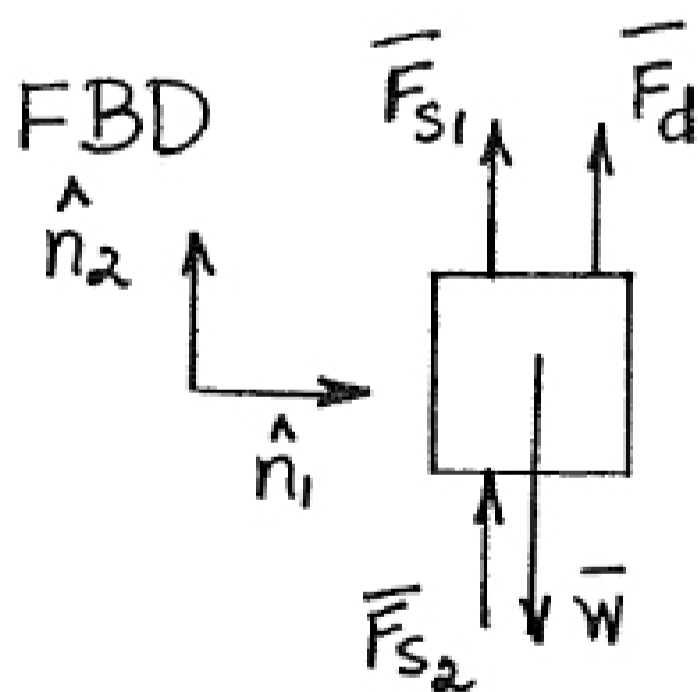
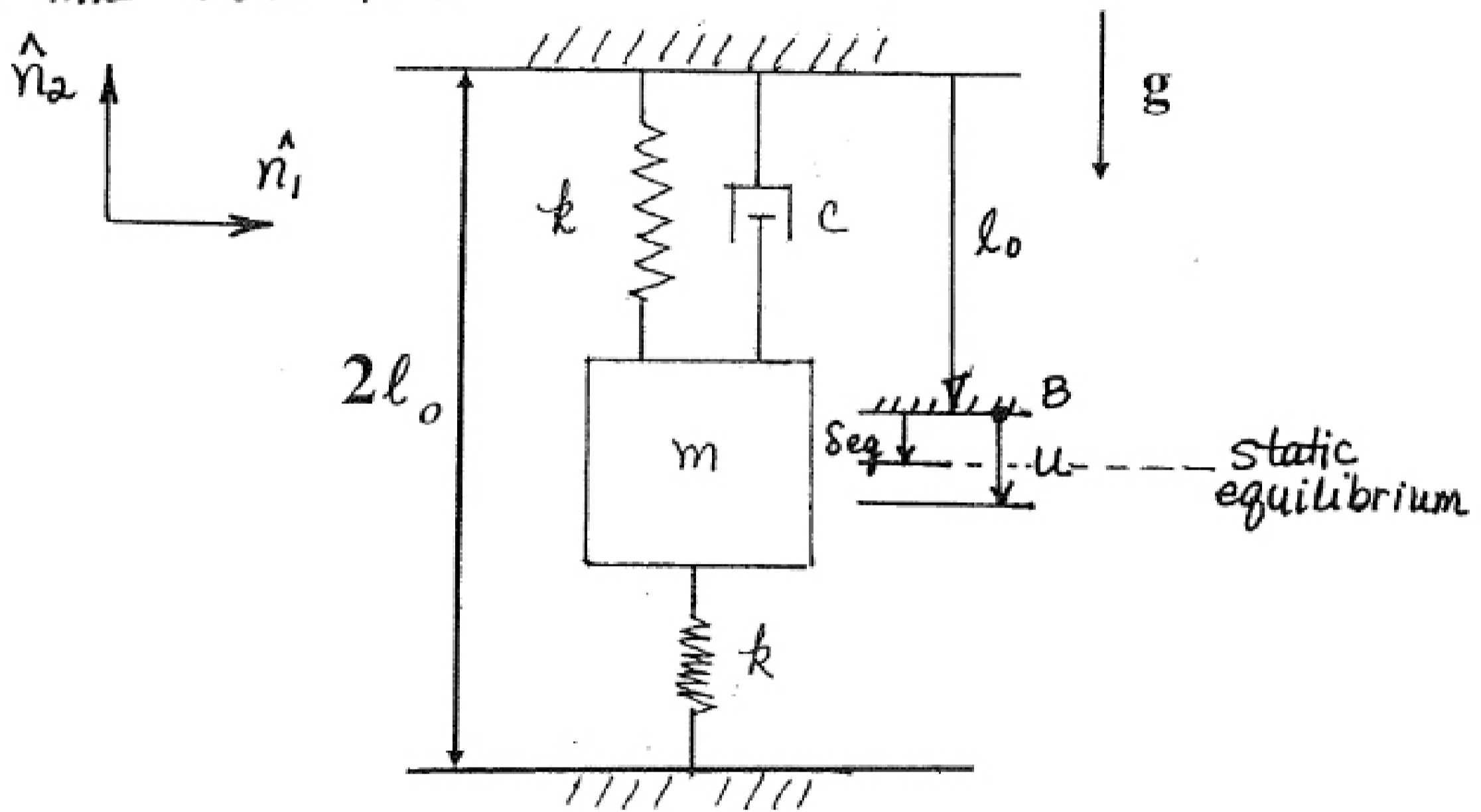
Please read the problems carefully.  
Write clearly and use diagrams when necessary.

(30 points)

**Problem 1:** In the system below, a particle of mass  $m$  can move in a vertical plane. Define  $u$  as the position of the particle from the unstretched position of the spring, i.e.  $l_0$ . The static deflections of each spring is  $\delta_{eq}$ .

If  $u$  is the variable of interest, derive the equation of motion.

Define  $\hat{n}$ : inertial



force models Spring deformation

$$F_{s1} = F_{s2} = ku$$

$$F_d = c\dot{u}$$

$$W = mg$$

Kinematics  $\vec{r}^{BP} = -u\hat{n}_2$

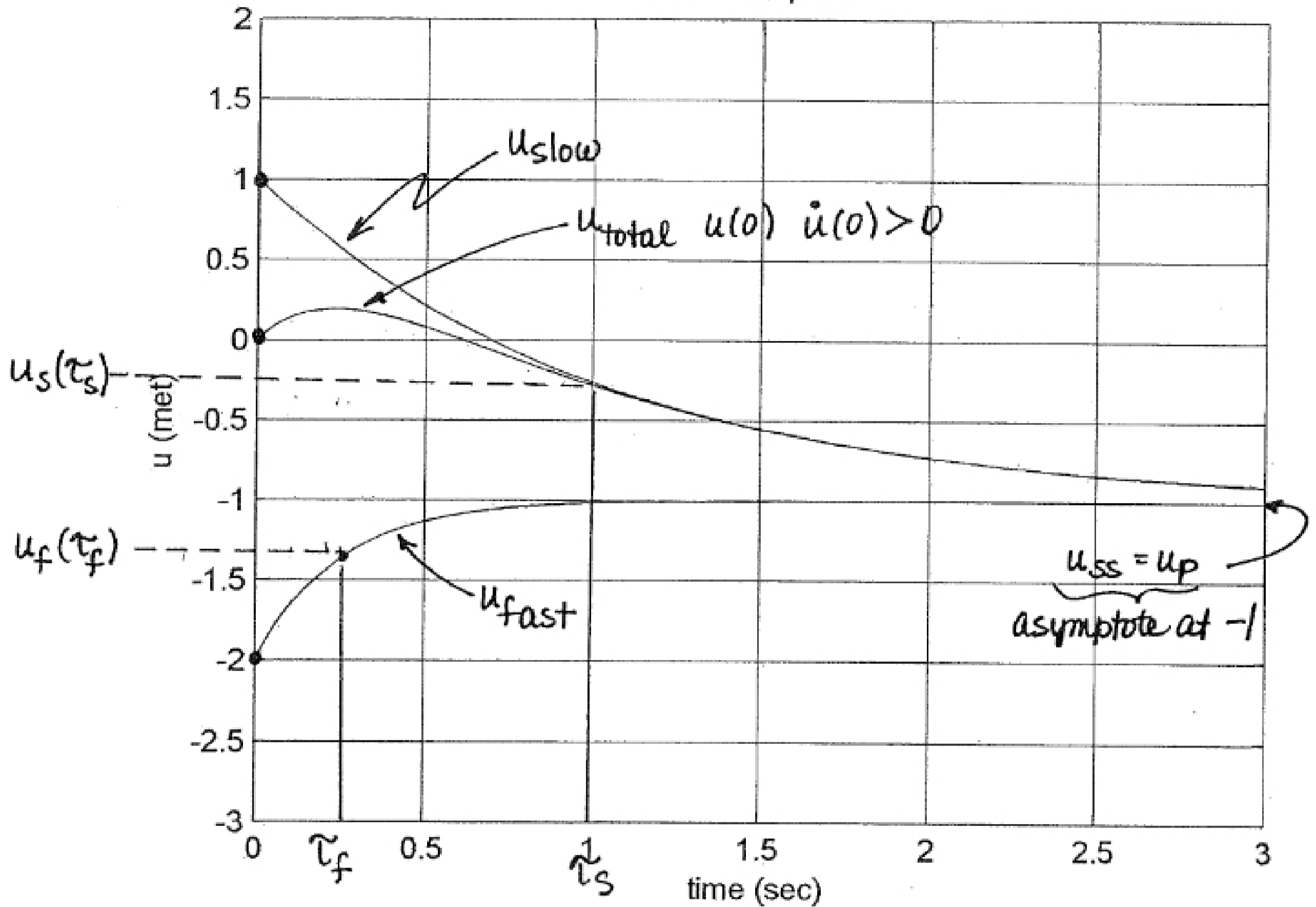
$${}^n\vec{v}^P = -\dot{u}\hat{n}_2$$

$${}^n\vec{a}^P = -\ddot{u}\hat{n}_2$$

$$\hat{n}_2: 2ku + c\dot{u} - mg = -m\ddot{u}$$

$$\boxed{\ddot{u} + \frac{c}{m}\dot{u} + \frac{2k}{m}u = g}$$

### Time Response



(ii)  $u_{fast} = -e^{-4t} - 1 \text{ met}$        $u_f(0) = -2$

$u_{slow} = 2e^{-t} - 1 \text{ met}$        $u_s(0) = +1$

(iii)  $\tau_f = \frac{1}{|s|} = \frac{1}{4} \text{ sec}$        $\tau_s = \frac{1}{|s|} = 1 \text{ sec}$

$u_f(\tau_f) \approx -\frac{1}{3} - 1 \approx -\frac{4}{3}$        $u_s(\tau_s) \approx \frac{1}{3}(2) - 1 \approx -\frac{1}{3}$

(iv)  $u_{steady-state} = u_p = -1$