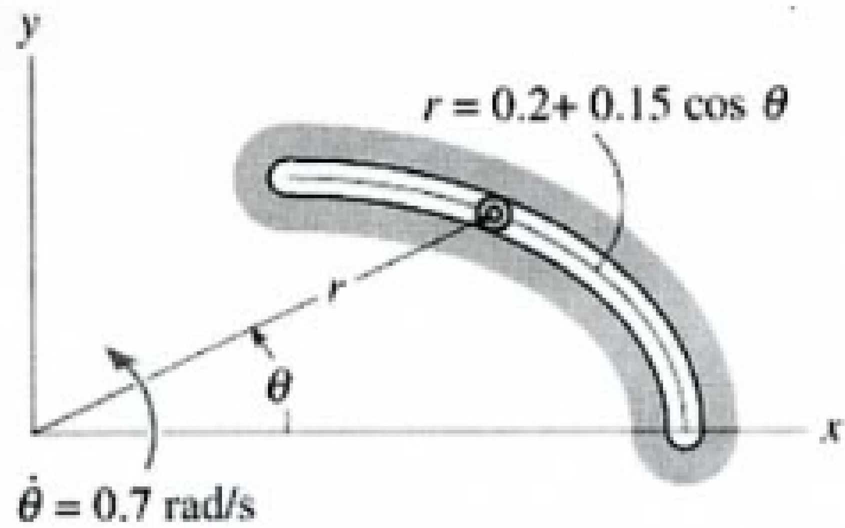
Sample Test 1 ME 201

1. (18 pts) The pin follows the path described in polar coordinates as shown where r is in meters. Determine the magnitude of the velocity and acceleration of the pin at the instant when the radial line is at $\theta = 30°$, angular velocity of the radial line is 0.7 rad/s, and the angular acceleration of the radial line is 0.5 rad/s^2. The size of the pin is negligible.



$$r = (0.2 + 0.15\cos\theta)\ \text{m}$$

$$\dot{r} = \frac{dr}{d\theta}\frac{d\theta}{dt} = (-0.15\sin\theta)(\dot{\theta})$$

$$\ddot{r} = (-0.15\cos\theta)(\dot{\theta})^2 + (-.15\sin\theta)(\ddot{\theta})$$

when $\theta = 30°$, $\dot{\theta} = 0.7\ \frac{\text{rad}}{\text{sec}}$, $\ddot{\theta} = 0.5\ \text{rad/sec}^2$

$r = 0.3299\ \text{m}$, $\dot{r} = -0.0525\ \frac{\text{m}}{\text{sec}}$, $\ddot{r} = -0.1012\ \frac{\text{m}}{\text{sec}^2}$

$$\bar{v} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta = -0.0525\,\hat{u}_r + (0.3299)(.7)\,\hat{u}_\theta$$

$$\bar{v} = (-0.0525\,\hat{u}_r + 0.2309\,\hat{u}_\theta)\ \text{m/sec}$$

$$\therefore\ v = 0.2368\ \text{m/sec}$$

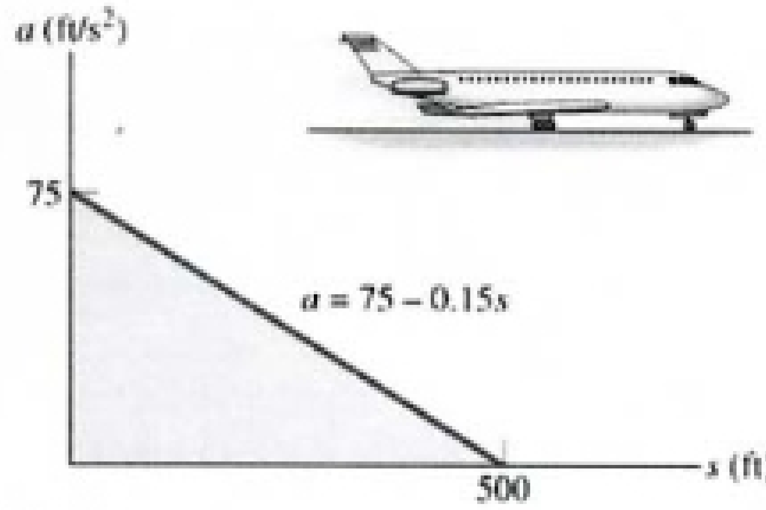
$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{u}_\theta$$

$$= (-0.1012 - 0.3299(.7)^2)\,\hat{u}_r + (0.3299(.5) + 2(-.0525)(.7))\,\hat{u}_\theta$$

$$\bar{a} = (-0.2629\,\hat{u}_r + 0.09145\,\hat{u}_\theta)\ \text{m/sec}^2$$

$$\therefore\ a = 0.2784\ \text{m/sec}^2$$

2. (20 pts) The jet aircraft starts from rest at position $s = 0$ and initiates takeoff, accelerating down the runway according to $a = 75 - 0.15\,s$ as shown, where position s is in feet and a is in ft/sec^2. Find the speed v as a function of position for the position for $s = 0$ to 500 ft. Evaluate the speed and time when $s = 250$ ft and $s = 500$ ft. Find the engine thrust required to produce the accelerations at these two locations if the aircraft weighs 200,000 lb and the drag force is $6.5\,v^2$ (where the force is in pounds and the speed v is in ft/sec) and rolling resistance is negligible.



Fn of Position:

$$a = 75 - 0.15\,s$$

$$v = \pm\sqrt{v_0^2 + 2\int_0^s a\,ds}$$

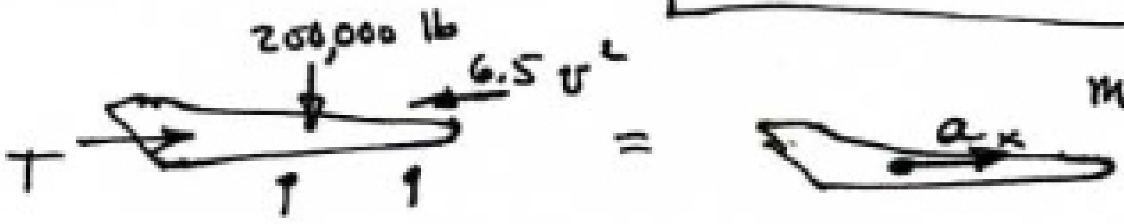
$$= \pm\sqrt{0 + 2\int_0^s (75 - 0.15\,s)\,ds}$$

$$\boxed{v = +\sqrt{150\,s - 0.15\,s^2}\ \frac{ft}{sec}}$$

see next page for time

when $s_1 = 250$ ft, $v_1 = 167.7$ ft/sec, $a = 37.5$ ft/sec^2

$s_2 = 500$ ft, $v_2 = 193.6$ ft/sec, $a = 0$



$$m = \frac{200\,000\ lb}{32.2\ ft/sec^2}$$

$$\rightarrow \Sigma F_x = m a_x$$

$$(T - 6.5\,v^2) = \frac{200{,}000}{32.2}\,a_x$$

$$T = 6211\,a_x + 6.5\,v^2$$

when $s_1 = 250$ ft: $T_1 = 415{,}721$ lb

$s_2 = 500$: $T_2 = 243{,}626$ lb

$$t = t_0 + \int_0^s \frac{ds}{v}$$

$$= 0 + \int_0^s (150\,s - 0.15\,s^2)^{-1/2}\,ds$$

from TI-89

$$\boxed{t = 2.582 \sin^{-1}(0.002(s-500)) + 4.056} \text{ sec}$$

when $s_1 = 250$ $\quad t_1 = 2.704$ sec

$s_2 = 500$ $\quad t_2 = 4.056$ sec