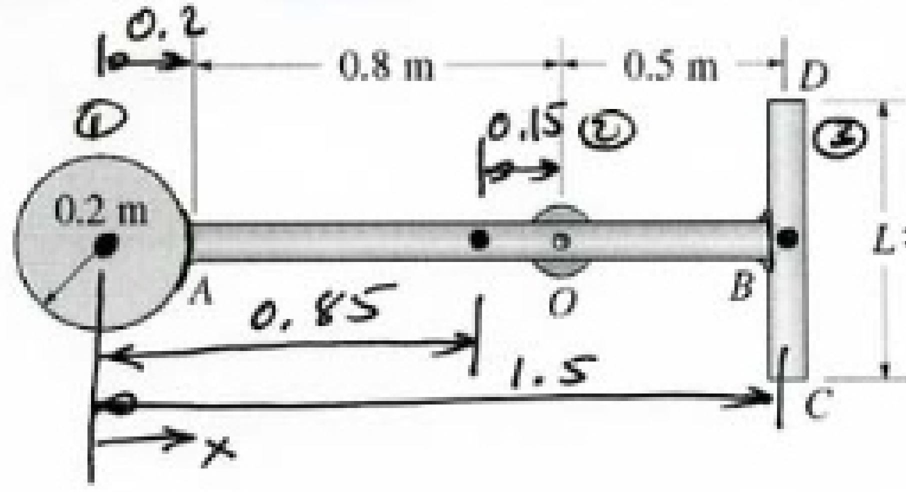
6. (20 pts) The assembly consists of two uniform thin rods, each having a mass per unit length of 2 kg/m, and a thin circular plate having a mass of 6 kg. The three parts are welded together and supported at a pin at "O". Let the length "L" be 0.4 m. Neglect the small fillets where the pieces are welded together. (a) Find the center of gravity of the assembly. (b) Find the mass moment of inertia about an axis perpendicular to the paper through the support at point "O". (c) Find the mass moment of inertia about an axis perpendicular to the paper through the center of gravity you found in part (a).



(a) $$m = 6\text{ kg} + \left(2\tfrac{\text{kg}}{\text{m}}\right)(1.3\text{ m}) + (0.4\text{ m})\left(2\tfrac{\text{kg}}{\text{m}}\right) = 9.4\text{ kg}$$

$$x_G = \frac{(6)(0) + (2.6)(0.85) + (0.8)(1.5)}{9.4} = \frac{3.41}{9.4} = \boxed{0.3628\text{ m}}$$

(b) $$I_O = \underbrace{\tfrac{1}{2}(6)(.2)^2 + 6(1.0)^2}_{\text{DISK}} + \underbrace{\tfrac{1}{12}(2.6)(1.3)^2 + 2.6(.15)^2}_{\text{long rod}}$$

$$+ \underbrace{\tfrac{1}{12}(0.8)(.4)^2 + 0.8(0.5)^2}_{\text{short rod}}$$

$$= 6.12 + 0.4247 + 0.2107 = \boxed{6.755\text{ kg-m}^2}$$

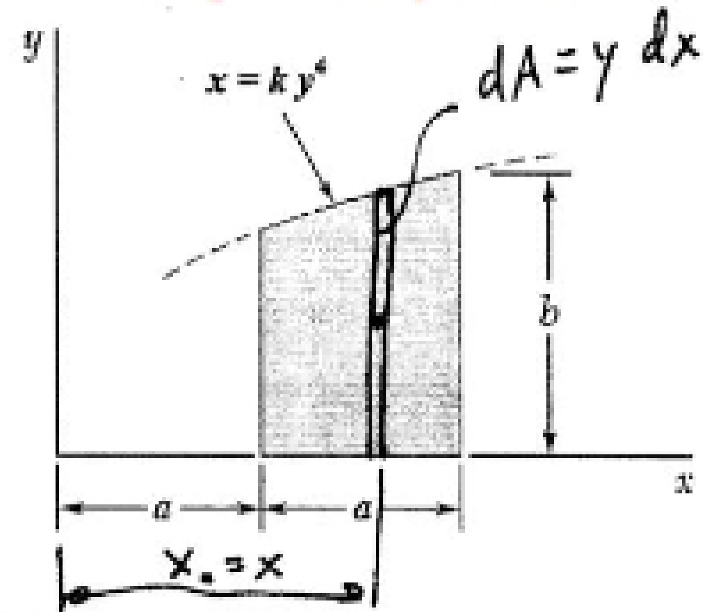
(c) $$I_O = I_G + m d^2$$

$$6.755 = I_G + (9.4)(1.0 - 0.3628)^2 = I_G + 3.8166$$

$$\boxed{I_G = 2.939\text{ kg-m}^2}$$

Note: different exponent on y in this solution

1. (35 pts) First find the constant k in terms of a and b. Then: (a) Find the distance from the y-axis to the centroid of the shaded area. (b) Find the moment of inertia of the shaded area about the y-axis. (c) Find the volume of the body created by rotating the shaded area about the y-axis.



- when $x = 2a$, $y = b$ $\therefore$ $2a = kb^4$ (5)

$$k = 2a/b^4$$

- Area:

$$A = \int y\,dx = \int_a^{2a} \left(\frac{x}{k}\right)^{1/4} dx = \frac{1}{k^{1/4}} \frac{x^{5/4}}{5/4}\Bigg|_a^{2a}$$

(10)

$$= \frac{b}{(2a)^{1/4}} \frac{4}{5}\left((2a)^{5/4} - a^{5/4}\right) = 0.9273\,ab = A$$

- $\tilde{x}$:

$$\tilde{x} = \frac{\int x_\circ dA}{A} = \frac{1}{A k^{1/4}} \int_a^{2a} x^{5/4} dx = \frac{1}{A k^{1/4}} \frac{x^{9/4}}{9/4}\Bigg|_a^{2a} = \frac{1}{A} \frac{b}{(2a)^{1/4}} \frac{4}{9}\left((2a)^{9/4} - a^{9/4}\right)$$

(10)

$$= \frac{1}{A}\, 1.404\, ba^2 = \boxed{1.514\,a = \tilde{x}}$$

- I_y:

$$I_y = \int x_\circ^2 dA = \int_a^{2a} \frac{x^{9/4}}{k^{1/4}} dx = \frac{b}{(2a)^{1/4}} \frac{x^{13/4}}{13/4}\Bigg|_a^{2a} = \frac{b}{(2a)^{1/4}} \frac{4}{13}\left((2a)^{13/4} - a^{13/4}\right)$$

(5)

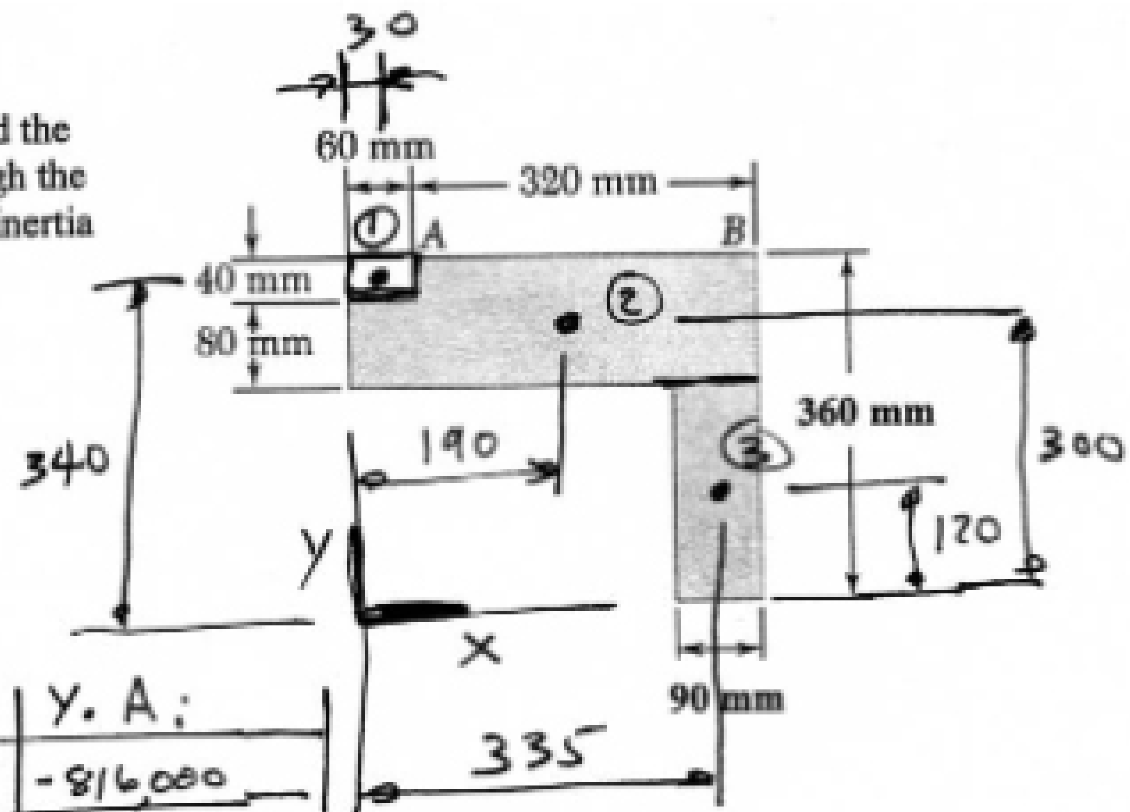
$$\boxed{I_y = 2.203\, ba^3}$$

- $Vol = 2\pi\,\tilde{x}\,A = 2\pi\,(1.514\,a)(.9273\,ab)$

$$\boxed{Vol = 8.821\,a^2 b}$$

(5)

3. (40 pts) (a) Find the centroid of the area. (b) Find the moment of inertia about the horizontal axis through the bottom edge of the area. (c) Find the moment of inertia about the horizontal axis through the centroid.



	x_i	y_i	A_i	$x_i A_i$	$y_i A_i$
1	30	340	-2,400	-72,000	-816,000
2	190	300	45,600	8,664,000	13,680,000
3	335	120	21,600	7,236,000	2,592,000
Σ			64,800	15,828,000	15,456,000

(10) $\bar{x} = \dfrac{\Sigma x_i A_i}{A} = \boxed{244.3 \text{ mm} = \bar{x}}$

(10) $\bar{y} = \dfrac{\Sigma y_i A_i}{A} = \boxed{238.5 \text{ mm} = \bar{y}}$

$$I_x = -\left(\tfrac{1}{12}(60)(40)^3 + 2{,}400(340)^2\right) \qquad -277{,}760{,}000$$
$$+\left(\tfrac{1}{12}(380)(120)^3 + 45{,}600(300)^2\right) \qquad +4{,}158{,}720{,}000$$
$$+\left(\tfrac{1}{12}(90)(240)^3 + 21{,}600(120)^2\right) \qquad +414{,}720{,}000$$

(10) $\boxed{I_x = 4.296 \times 10^9 \text{ mm}^4}$

$$I_x = I_{\bar{x}} + Ad^2 = I_{\bar{x}} + 64800(238.5)^2$$

(10) $\boxed{I_{\bar{x}} = 6.100 \times 10^8 \text{ mm}^4}$