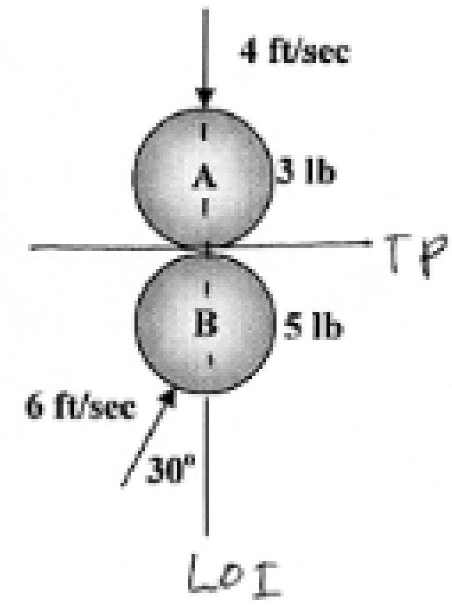
ME 201 **Test #5** **Fall 2006** Name (print): KEY

Section ________ Table ________ Group ________

1. (30 pts) Two spheres are translating with the velocities shown immediately before they collide. The weights are shown. The coefficient of restitution between the two materials is 0.60. The contact surfaces may be considered to be frictionless. Find the velocity vectors of each sphere immediately after the impact is complete. Find the percentage of loss in kinetic energy due to the impact.



Tangent Direction: (no Friction)

$\xrightarrow{+}$ $\quad v_{AT_1} = \boxed{0 = v_{AT_2}}$

$v_{BT_1} = \boxed{3\text{ ft/sec} \rightarrow \; = v_{BT_2}}$

Material: Normal Direction ONLY!

$+\uparrow \quad e = 0.6 = \dfrac{v_{AN2} - v_{BN2}}{v_{BN1} - v_{AN1}} = \dfrac{v_{AN2} - v_{BN2}}{6\cos 30 - (-4)}$

CONSISTENT

$\therefore \; v_{AN2} - v_{BN2} = 5.518$

Normal Direction: Cons. Momentum

$+\uparrow \quad m_A v_{AN1} + m_B v_{BN1} = m_A v_{AN2} + m_B v_{BN2}$

$\frac{3}{32.2}(-4) + \frac{5}{32.2}(6\cos 30) = \frac{3}{32.2} v_{AN2} + \frac{5}{32.2} v_{BN2}$

$3 v_{AN2} + 5 v_{BN2} = 13.98$

Solve: $v_{AN2} = 5.196$ ft/sec $\uparrow$

$v_{BN2} = -0.3217 = 0.3217$ f/sec $\downarrow$

$\therefore \; \boxed{v_{A2} = 5.196 \text{ ft/sec} \uparrow}$

$\boxed{v_{B2} = (3\hat{i} - 0.3217\hat{j}) \text{ ft/sec}}$ $\quad {}^{y}\llcorner_{x}$ $\quad \therefore v_{B2} = 3.017$ ft/sec

Energy Loss:

$T_1 = \frac{1}{2}\frac{3}{32.2}(4)^2 + \frac{1}{2}\frac{5}{32.2}(6)^2 = 3.5404$

$T_2 = \frac{1}{2}\frac{3}{32.2}(5.196)^2 + \frac{1}{2}\frac{5}{32.2}(3.017)^2 = 1.9644$

$\boxed{\dfrac{\Delta T}{T_1} = 44.5\%}$

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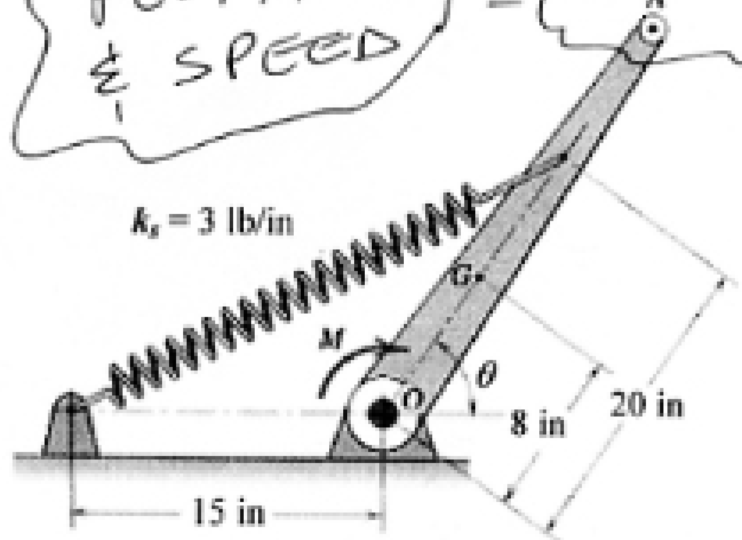
Section ________ Table ________ Group ________

POSITION & SPEED = WORK-ENERGY

2. (30 pts) The system rotates in a vertical plane and is at rest in the vertical position when $\theta = 90°$. The 12-lb nonuniform bar OA has a center of gravity at G as shown and a 6-in radius of gyration about an axis at G perpendicular to the plane of motion. The pin bearing at O is frictionless. The linear spring is unstretched in the initial at rest position. The spring has a stiffness of 3 lb/in, or 36 lb/ft. A constant couple M is applied to cause the system to rotate clockwise. Find the magnitude of M such that when the bar is horizontal ($\theta = 0°$) the angular speed will be 4 rad/sec.

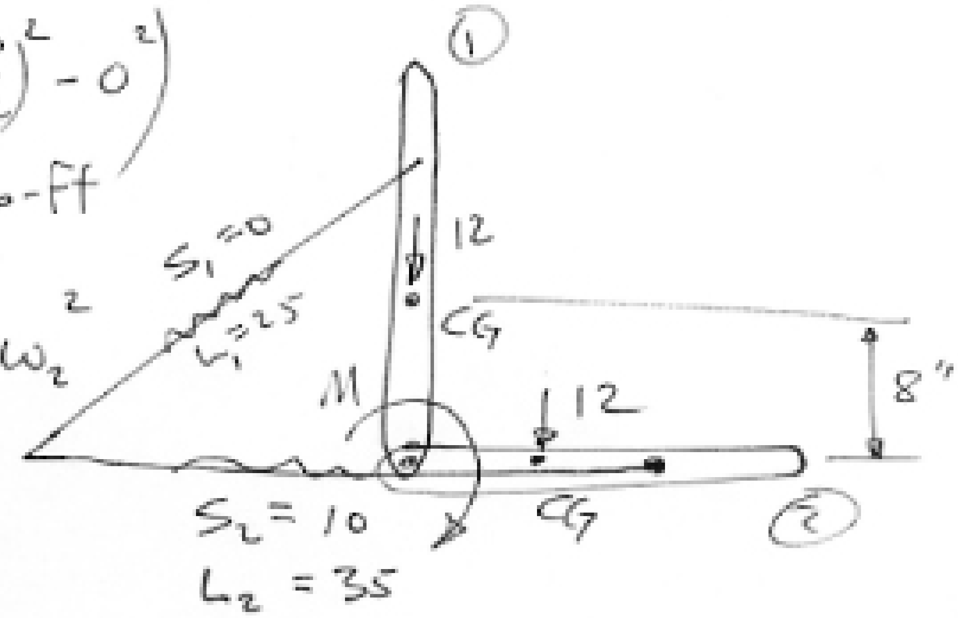


$1 \to 2$:

$T_1 = 0$

$$U_{1\to2} = 12\left(\frac{8}{12}\right) + M\left(\frac{\pi}{2}\right) - \frac{36}{2}\left(\left(\frac{10}{12}\right)^2 - 0^2\right)$$

$$= \left(8 + M\frac{\pi}{2} - 12.5\right) \text{ lb-ft}$$



$$T_2 = \frac{1}{2}\,\frac{12}{32.2}\,v_{G_2}^2 + \frac{1}{2}\left(\frac{12}{32.2}\left(\frac{6}{12}\right)^2\right)\omega_2^2$$

kinematics $v_{G_2} = \frac{8}{12}\,\omega_2$

$$T_2 = 0.1294\,\omega_2^2$$

$$\therefore \quad T_1 + U_{1\to2} = T_2$$

$\omega_2 = 4$

$$0 + \left(M\frac{\pi}{2} - 4.5\right) = 0.1294\,\omega_2^2 = 2.0704$$

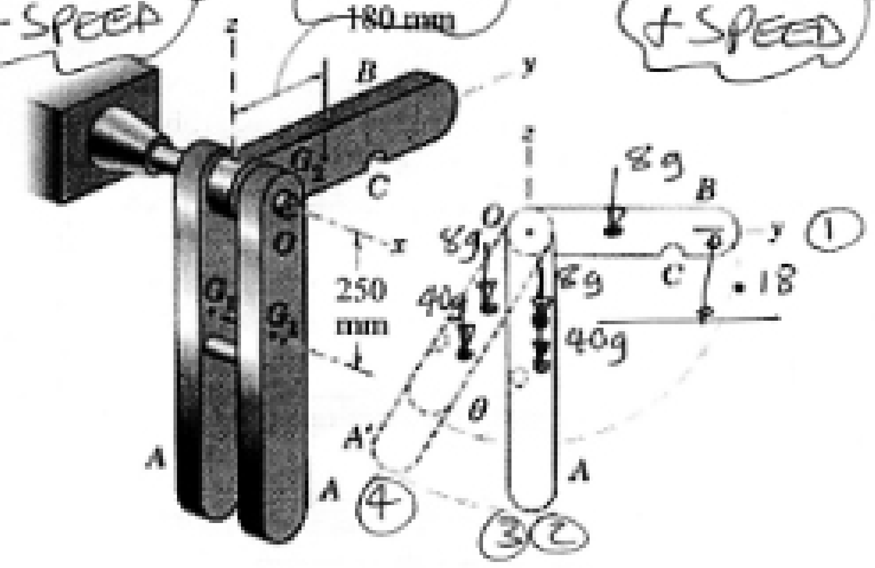
$$\boxed{\begin{aligned} M &= 4.183 \text{ lb-ft} \\ &= 50.19 \text{ lb-in} \end{aligned}}$$

ME 201 **Test #5** **Fall 2006** Name (print): ① → ② ② → ③ ③ → ④

Section Table Group

POSITION + SPEED IMPACT POSITION + SPEED

3. (40 pts) The three bars can rotate about the x-axis on a frictionless bearing. The two bars A are connected so as to move together as one. Each of the two bars A has a mass of 20 kg and radius of gyration about a parallel x-axis through their CG of 200 mm. The shorter bar B has a mass of 8 kg and radius of gyration about a parallel x-axis through its CG of 150 mm. The respective CG locations are shown. The bars are at rest when bar B is released from its original horizontal position as shown. When bar B is vertical, the notch at C on bar B impacts the connecting pin between the two A bars and attaches to the pin (i.e., the coefficient of restitution is zero). After impact the three bars move as one. Calculate the maximum angle θ to which they rotate as they come to a stop for an instant.



$$I_{G_B} = 8(.15)^2 = 0.18 \text{ kg-m}^2$$

$$I_{G_A} = 2(20)(.2)^2 = 1.6 \text{ kg-m}^2$$

kinematics: $v_{BG} = .18\,\omega_B$

① → ② before impact

$$T_1 + U_{1\to2} = T_2$$

$$0 + 8(9.81)(.18) = \tfrac{1}{2}(.18)\,\omega_{B2}^2 + \tfrac{1}{2}\,8(.18\,\omega_{B2})^2$$

$$\therefore \omega_{B2} = 8.02 \text{ rad/sec}$$

$$\therefore v_{GB2} = 1.4436 \tfrac{m}{sec}$$

② → ③ during impact

weight = non-impulsive

reactions at O = impulsive

$\therefore$ select pt. O & use system angular impulse-mom.

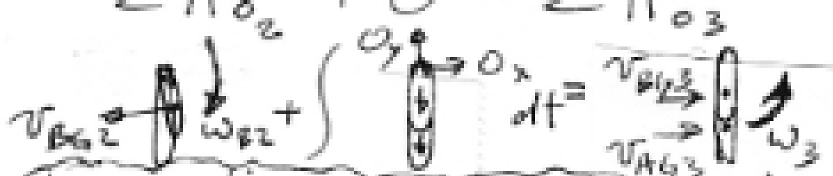
NOTE: using y, x

$$\Sigma \bar{H}_{O2} + 0 = \Sigma \bar{H}_{O3}$$

$$\bar{H}_{O2} = I_{GB}\,\bar{\omega}_{B2} + \bar{r}_{GB/O} \times m_B \bar{v}_{B2}$$

$$= .18(-8.02)\hat{k} + (-.18\hat{j}) \times 8(-1.4436\hat{i})$$

$$\bar{H}_{O2} = -3.522\,\hat{k} \text{ kg-m}^2\text{/sec}$$



kinematics: $v_{AG} = .25\,\omega_A$

$e = 0 \therefore \omega_{A3} = \omega_{B3} = \omega_3$

assumes ω_3 $+\hat{k}$ and we expect

$$\therefore \bar{H}_{O3} = (0.18 + 1.6)\,\omega_3\hat{k} + (-.18\hat{j}) \times 8(+.18\,\omega_3\hat{i}) + (-.25\hat{j} \times 40(.25\,\omega_3\hat{i}))$$

$$\bar{H}_{O3} = 4.5392\,\omega_3\,\hat{k}$$

$$\therefore -3.522 + 0 = 4.5392\,\omega_3$$

$$\omega_3 = -0.7759 = 0.7759 \tfrac{\text{rad}}{\text{sec}}$$