Name (print): KEY

Section ________ Table ________

Honor Code: *I have neither given nor received unauthorized aid on this test*

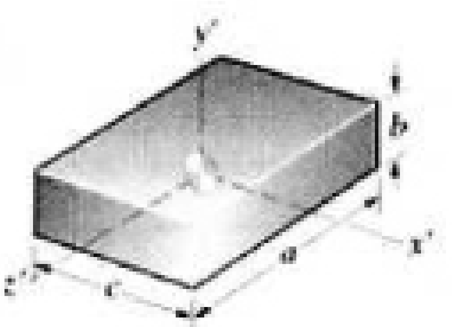
Signature: ______________________________

ME 201 Test #3

Fall 2009

NOTES:

- ***Sign and provide identifying info on every sheet.***
- *Use proper **vector notation** in all cases where vectors are used.*
- *In cases involving Newton's Laws, you are **REQUIRED to draw** complete and correct **FBDs** and when the problem is dynamics, you must also draw complete and correct **KDs**. Then use these to develop your governing equations.*
- *If you use you **calculator to perform any calculus** that might involve trig or similar functions, set your calculator to the **radian mode** first.*
- *In all cases, remember to show results with **magnitude, direction, units** and put your **answer in a box**.*



Rectangular prism

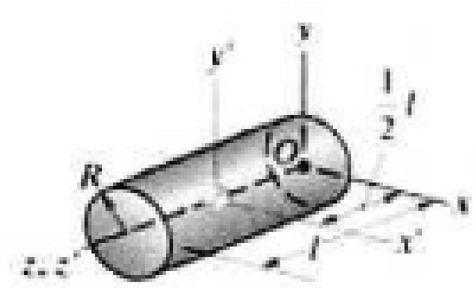
Volume $= abc$

$$I_{x'\,\text{axis}} = \frac{1}{12}m(a^2 + b^2), \qquad I_{y'\,\text{axis}} = \frac{1}{12}m(a^2 + c^2),$$

$$I_{z'\,\text{axis}} = \frac{1}{12}m(b^2 + c^2),$$

Volume $= \pi R^2 l$

$$I_{x\,\text{axis}} = I_{y\,\text{axis}} = m\left(\frac{1}{3}l^2 + \frac{1}{4}R^2\right), \qquad I_{z\,\text{axis}} = \frac{1}{2}mR^2,$$



Circular cylinder

$$I_{x'\,\text{axis}} = I_{y'\,\text{axis}} = m\left(\frac{1}{12}l^2 + \frac{1}{4}R^2\right), \qquad I_{z'\,\text{axis}} = \frac{1}{2}mR^2,$$

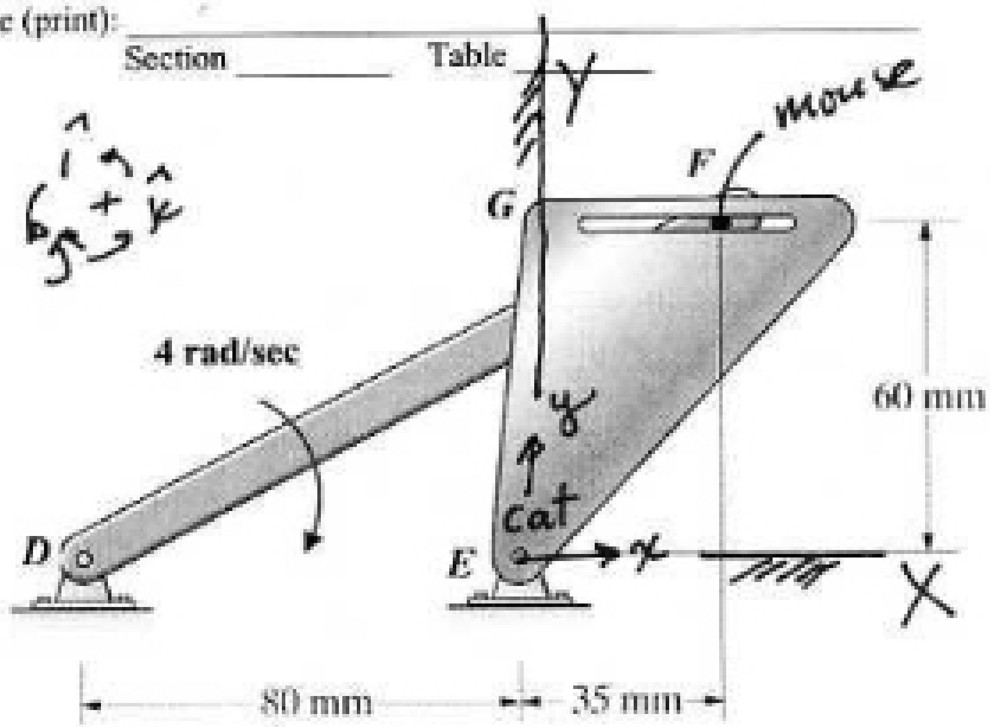
For a uniform slender bar with radius very small compared to length, these simplify to the familiar values about CG and end, respectively: $I_{Gx} = m\,l^2/12$ and $I_{end\text{-}x} = m\,l^2/3$, and z-axis values go to zero.

For a uniform circular disk with thickness very small compared to radius, these simplify to the familiar values about CG: $I_{Gx} = m\,R^2/4$ and $I_{Gz} = m\,R^2/2$.

ME 201 **Test #3** **Fall 2009** Name (print): ____________________

Section ________ Table ____

1. (33 pts) Bar DF has a constant angular velocity of 4 rad/sec in the clockwise direction. The small pin at F is attached to bar DF and slides in the slot in body EG. At the instant shown, the slot in body EG is horizontal. At this instant,
   1. find the **angular velocity of body EG,** and
   2. find the **angular acceleration of body EG.**



Using D-F:

VEL: $\bar{v}_F = \cancel{\bar{v}_D}^{\,0} + \bar{\omega}_{DF} \times \bar{r}_{F/D}$

$= \bar{0} + (-4\hat{k}) \times (115\hat{i} + 60\hat{j}) = (240\hat{i} - 460\hat{j})$ mm/sec

ACCEL: $\bar{a}_F = \cancel{\bar{a}_D}^{\,0} + \cancel{\bar{\alpha}_{DF}}^{\,0} \times \bar{r}_{F/D} - \omega_{DF}^2\, \bar{r}_{F/D} = (-1840\hat{i} - 960\hat{j})$ mm/sec²

Using F as mouse moving on EG. Let cat be at E & align XYZ & xyz $\therefore \hat{i} = \hat{I} \quad \hat{j} = \hat{J} \quad \hat{k} = \hat{K}$

VEL: $\bar{v}_F = \cancel{\bar{v}_E}^{\,\bar{0}} + \bar{v}_{F/E} + \bar{\Omega}_{EG} \times \bar{r}_{F/E}$

$240\hat{i} - 460\hat{j} = \bar{0} + v_{F/E}\,\hat{i} + \Omega_{EG}\,\hat{k} \times (35\hat{i} + 60\hat{j})$

$= (v_{F/E} - 60\,\Omega_{EG})\,\hat{i} + 35\,\Omega_{EG}\,\hat{j}$

$\hat{i}: \; 240 = v_{F/E} - 60\,\Omega_{EG}$

$\hat{j}: \; -460 = 35\,\Omega_{EG}$

$\therefore \boxed{\Omega_{EG} = -13.14 = 13.14 \frac{\text{rad}}{\text{sec}}}$

$v_{F/E} = -548.4 \frac{\text{mm}}{\text{sec}}$

ACCEL: $\bar{a}_F = \cancel{\bar{a}_E}^{\,\bar{0}} + \bar{a}_{F/E} + \dot{\bar{\Omega}}_{EG} \times \bar{r}_{F/E} - \Omega_{EG}^2\, \bar{r}_{F/E} + 2\bar{\Omega}_{EG} \times \bar{v}_{F/E}$

$-1840\hat{i} - 960\hat{j} = a_{F/E}\,\hat{i} + \dot{\Omega}_{EG}\,\hat{k} \times (35\hat{i} + 60\hat{j}) - (-13.14)^2(35\hat{i} + 60\hat{j})$

$+ 2(-13.14\,\hat{k}) \times (-548.4\,\hat{i})$

$\hat{i}: \; -1840 = a_{F/E} - 60\,\dot{\Omega}_{EG} - 6043.1$

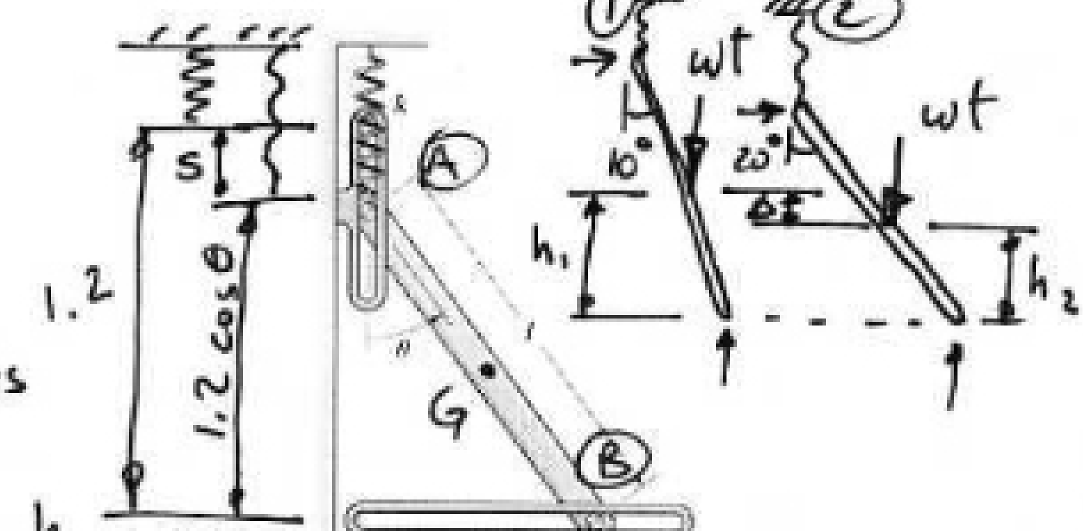
$\hat{j}: \; -960 = 35\,\dot{\Omega}_{EG} + 4052.4$

$\therefore \boxed{\dot{\Omega}_{EG} = -143.2 = 143.2 \frac{\text{rad}}{\text{sec}^2}}$

$a_{F/E} = -4389.6 \frac{\text{mm}}{\text{sec}^2}$

USE WORK-ENERGY

2. (33 pts) The 4-kg uniform bar is 1.2 m long. The pins at each end slide in the stationary frictionless slots. The spring constant $k = 180$ N/m. The spring is unstretched when the bar is vertical ($\theta = 0°$). The system is released from rest when $\theta = 10°$. Find the **angular velocity** of the bar when $\theta = 20°$.



$s = 1.2(1 - \cos\theta)$ = stretch as fn of θ

@ $\theta = 10°$ $\quad s_1 = 0.01823$ m

@ $\theta = 20°$ $\quad s_2 = 0.07237$ m

$h_1 = 0.5909$

$h_2 = 0.5638$

$\Delta = 0.02707$ m ↓

kinetics:

$\cancel{T_1}^{\,0} + U_{1\to 2} = T_2 \; : \quad 4(9.81)(0.02707) - \frac{180}{2}\left(.07237^2 - .01823^2\right) = T_2$

$0.62077 \text{ N·m} = \frac{1}{2} m v_{G2}^2 + \frac{1}{2} I_G \omega_2^2$

kinematics: eq 7-7 at position 2 :

we observe $\bar{v}_A = -v_A \hat{j} \quad \bar{v}_B = v_B \hat{i}$ $\quad$ using A + B

we have $\bar{v}_A = \bar{v}_B + \bar{\omega}_2 \times \bar{r}_{A/B}$

$-v_A \hat{j} = v_B \hat{i} + \omega_2 \hat{k} \times \left(1.2(-\sin 20\,\hat{i} + \cos 20\,\hat{j})\right)$

$= \left(v_B - 1.2\cos 20\,\omega_2\right)\hat{i} + \left(-1.2\sin 20\,\omega_2\right)\hat{j}$

$\hat{i}: \quad 0 = v_B - 1.2\cos 20\,\omega_2 \qquad v_B = 1.1276\,\omega_2 \rightarrow$

$\hat{j}: \quad -v_A = -1.2\sin 20\,\omega_2 \qquad v_A = 0.4104\,\omega_2 \downarrow$

Now using B + G (could use A + G just as well):

$\bar{v}_G = \bar{v}_B + \bar{\omega}_2 \times \bar{r}_{G/B} = 1.1276\,\omega_2 \hat{i} + \omega_2 \hat{k} \times \left(0.6(-\sin 20\,\hat{i} + \cos 20\,\hat{j})\right)$

$\therefore \bar{v}_G = \left(1.1276\,\omega_2 - .6\cos 20\,\omega_2\right)\hat{i} - \left(.6\sin 20\,\omega_2\right)\hat{j}$

$\bar{v}_G = 0.56378\,\omega_2 \hat{i} - 0.20521\,\omega_2 \hat{j} \quad \therefore v_G = 0.600\,\omega_2$

Substitute in kinetics:

$0.62077 = \frac{1}{2} 4(.600\,\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12} 4 (1.2)^2\right)\omega_2^2 \quad \therefore \boxed{\omega_2 = 0.8041 \frac{\text{rad}}{\text{sec}} \circlearrowleft}$