wksht 8 solns

1. a $f(n) = \frac{n+1}{3n-1}$

$a_1 = f(1) = \frac{1+1}{3-1} = \frac{2}{2} = 1$

$a_2 = f(2) = \frac{2+1}{6-1} = \frac{3}{5}$

$a_3 = f(3) = \frac{3+1}{9-1} = \frac{4}{8} = \frac{1}{2}$

$a_4 = f(4) = \frac{4+1}{12-1} = \frac{5}{11} \approx .4545$

$a_5 = f(5) = \frac{5+1}{15-1} = \frac{6}{14} = \frac{3}{7} \approx .4286$

$a_{500} = f(500) = \frac{501}{1499} \approx .3342$

$a_{1000} = f(1000) = \frac{1001}{2999} \approx .3338$

$\vdots$

$a_{10{,}000} = \frac{10{,}001}{29{,}999} \approx .333378$

$\vdots$

getting closer to $.333\overline{3} = \frac{1}{3}$

i. sequence is $\{1, 3/5, 1/2, 5/11, 3/7, \ldots, \frac{501}{1499}, \ldots, \frac{1001}{2999}, \ldots\}$

ii. $\lim_{n\to\infty} f(n) = \lim_{n\to\infty} \frac{n+1}{3n-1} = \frac{1}{3}$

iii. bdd above, bdd below, strictly decreasing, convergent.

b. $f(n) = \frac{(-1)^n}{2n}$

$a_1 = f(1) = \frac{(-1)^1}{2} = -\frac{1}{2}$

$a_2 = f(2) = \frac{(-1)^2}{4} = \frac{1}{4}$

$a_3 = f(3) = \frac{(-1)^3}{6} = -\frac{1}{6}$

$a_4 = f(4) = \frac{(-1)^4}{8} = 1/8$

$a_5 = f(5) = \frac{(-1)^5}{10} = \frac{-1}{10} = -.1$

$a_{500} = f(500) = \frac{(-1)^{500}}{1000} = \frac{1}{1000} = .001$

$a_{1000} = f(1000) = \frac{(-1)^{1000}}{2000} = \frac{1}{2000} = .0005$

$\vdots$

$a_{10000} = \frac{1}{20{,}000} = .00005$

$\vdots$

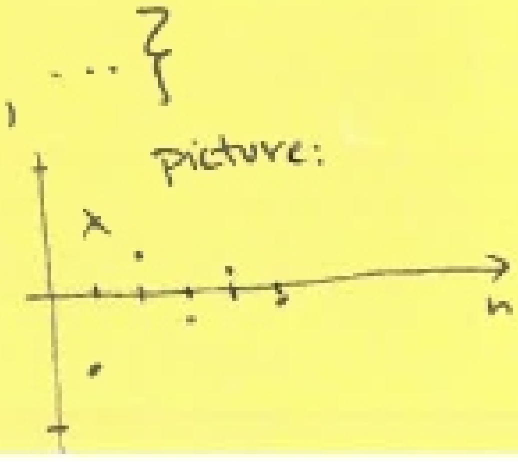
getting closer to 0.

i. seq is $\{-1/2, 1/4, -1/6, 1/8, -1/10, \ldots, 1/1000, \ldots, \frac{1}{2000}, \ldots\}$

ii. $\lim_{n\to\infty} \frac{(-1)^n}{2n} = 0$

iii. bdd above, bdd below, alternating, convergent

picture:



c. $a_n = \frac{-n}{4} + 3$

$a_1 = -\frac{1}{4} + 3 = \frac{11}{4}$

$a_2 = -\frac{2}{4} + 3 = \frac{5}{2}$

$a_3 = -\frac{3}{4} + 3 = \frac{9}{4}$

$a_4 = -1 + 3 = 2$

$a_5 = -\frac{5}{4} + 3 = \frac{7}{4}$

$a_{500} = \frac{-500}{4} + 3$

$= -125 + 3 = -122$

$a_{1000} = \frac{-1000}{4} + 3$

$= -250 + 3 = -247$

$\vdots$

$a_{10000} = \frac{-10000}{4} + 3 = -2497$

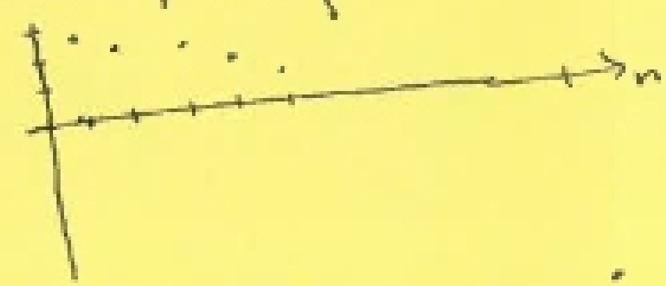
$\vdots$

keeps decreasing without bound

i seq is $\{ 11/4, 5/2, 9/4, 2, 7/4, \ldots, -122, \ldots, -247, \ldots \}$

ii $\lim_{n \to \infty} \frac{-n}{4} + 3 = -\infty$

picture:



iii strictly decreasing, bounded above, divergent

d. $a_n = -12$

$a_1 = -12$
$a_2 = -12$
$a_3 = -12$
$a_4 = -12$
$a_5 = -12$

$a_{500} = -12$

$a_{1000} = -12$

i. seq is $\{-12, -12, -12, -12, \ldots\}$

ii. $\lim_{n \to \infty} -12 = -12$

iii. bdd above, bdd below, convergent

e. $f(n) = (-1)^{n+1}(2n)$

$a_1 = (-1)^2 \cdot 2 = 2$
$a_2 = (-1)^3 \cdot 4 = -4$
$a_3 = (-1)^4 \cdot 6 = 6$
$a_4 = (-1)^5 \cdot 8 = -8$
$a_5 = (-1)^6 \cdot 10 = 10$

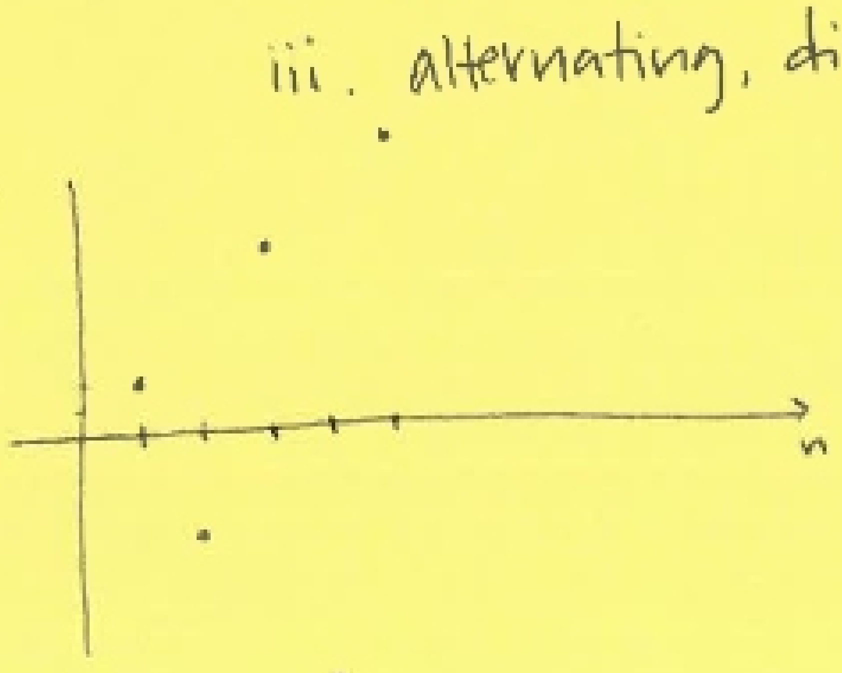
$a_{500} = -1000$

$a_{1000} = -2000$

i. $\{2, -4, 6, -8, 10, \ldots, -1000, \ldots, -2000, \ldots$

ii. $\lim_{n \to \infty} (-1)^{n+1}(2n)$ does not exist.

iii. alternating, divergent



2\. a

$$\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ -\frac{1}{2^2} & \frac{2}{3^2} & -\frac{3}{4^2} & \frac{4}{5^2} \end{array}$$

alternating so has $(-1)^n$.

numerator $= n$

denom $= (n+1)^2$

$$= \frac{(-1)^n n}{(n+1)^2}$$

b.

$$\frac{1}{1} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7} \quad \frac{5}{9}$$

numerator $= n$

denom $=$ odd #s $= 2n-1$

$$= \frac{n}{2n-1}$$