

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 20 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [20 pnts.] Disprove by counter example or Prove each of the following:
 - a. The sum of any rational number and any integer is rational.

show: $\forall r \in Q, \forall m \in Z, r + m \in Q$

PROOF:

Let $r \in Q$ and $m \in Z$ be arbitrary.

Since $r \in Q$, $\exists a, b \in Z$ where $b \neq 0, r = \frac{a}{b}$ by definition of rational

$r + m = \frac{a}{b} + m$ by substitution

$= \frac{a}{b} + \frac{mb}{b}$ by multiplying second term by $\frac{b}{b}$

$= \frac{a+mb}{b}$ by moving them over the common denominator

$a + mb \in Z$ because of closure of Z during the operations of $+$ and $*$

$b \in Z$ where $b \neq 0$ as defined above

therefore $r + m \in Q$ because it is a quotient of integers as required by the definition of rational.

$\forall r \in Q, \forall m \in Z, r + m \in Q$ by generalizing from the Generic Particular.

- b. For every integer n , $n^2 - n + 3 \equiv_2 1$

$\forall n \in Z, n^2 - n + 3 \equiv_2 1$

PROOF:

Let n be arbitrary in Z .

By the Quotient Remainder Theorem, we know that $\exists q \in Z, n = 2q + 0 \vee n = 2q + 1$

Case 1 ($n = 2q + 0$):

$n^2 - n + 3 = (2q)^2 - (2q) + 3$ by substitution

$n^2 - n + 3 = 4q^2 - 2q + 2 + 1$ by multiplication and addition

$n^2 - n + 3 = 2(2q^2 - q + 1) + 1$ by factoring

Since $2q^2 - q + 1 \in Z$ by closure of integers in multiplication and addition,

$\exists j \in Z, n^2 - n + 3 = 2j + 1$ by substitution.

$(n^2 - n + 3) - 1 = 2j$ by subtracting 1 from both sides.

$2 \mid [(n^2 - n + 3) - 1]$ by definition of divides.

$n^2 - n + 3 \equiv_2 1$ by definition of equivalence in a mod

Case 2 ($n = 2q + 1$):

$n^2 - n + 3 = (2q + 1)^2 - (2q + 1) + 3$ by substitution

$n^2 - n + 3 = 4q^2 + 4q + 1 - 2q - 1 + 3$ by multiplication

$n^2 - n + 3 = 4q^2 + 2q + 2 + 1$ by algebra

$n^2 - n + 3 = 2(2q^2 + q + 1) + 1$ by factoring

Since $2q^2 + q + 1 \in \mathbb{Z}$ by closure of integers in multiplication and addition

$\exists k \in \mathbb{Z}, n^2 - n + 3 = 2k + 1$ by substitution

$(n^2 - n + 3) - 1 = 2k$ by subtracting 1 from both sides.

$2 \mid [(n^2 - n + 3) - 1]$ by definition of divides.

$n^2 - n + 3 \equiv_2 1$ by definition of equivalence in a mod

Since the quotient remainder theorem tells us that these are the only two possibilities and both of these lead to the fact that $n^2 - n + 3 \equiv_2 1$, by dilemma we know that $n^2 - n + 3 \equiv_2 1$

$\forall n \in \mathbb{Z}, n^2 - n + 3 \equiv_2 1$ by generalizing from the Generic Particular.

2. [4 pts.] Write the standard factored form of 1050:

$$1050 = 2^1 * 3^1 * 5^2 * 7^1$$

3. [6 pts.] Use the unique factorization theorem and suppose that m is an integer such that

$$5 * 4 * 3 * 2 * m = 10 * 11 * 12 * 13$$

Circle Yes or No for each of the following: Yes means that this is something that must be true, No means it doesn't necessarily need to be true:

a)	$10 \mid m$	YES	(NO)
b)	$11 \mid m$	(YES)	NO
c)	$12 \mid m$	YES	(NO)
d)	$13 \mid m$	(YES)	NO
e)	$24 \mid m$	YES	(NO)
f)	$143 \mid m$	(YES)	NO