Math 199 Mock Exam 3 · 17 November, 2014

1. Suppose $\ln(x) = 3$, $\ln(y) = 7$, $\ln(z) = 9$, and $b = e^2$. Simplify $\log_b\left(\frac{x^2y}{z}\right)$.

$$\begin{aligned}
\log_b\left(\frac{x^2y}{z}\right) &= \log_b(x^2y) - \log_b(z) \\
&= \log_b(x^2) + \log_b(y) - \log_b(z) \\
&= 2\log_b(x) + \log_b(y) - \log_b(z) \\
&= 2\frac{\ln x}{\ln b} + \frac{\ln y}{\ln b} - \frac{\ln z}{\ln b} \\
&= 2\frac{\ln x}{\ln e^2} + \frac{\ln y}{\ln e^2} - \frac{\ln z}{\ln e^2} \\
&= 2\frac{\ln x}{2} + \frac{\ln y}{2} - \frac{\ln z}{2} \\
&= \ln x + \frac{\ln y}{2} - \frac{\ln z}{2} \\
&= 3 + 7/2 - 9/2 \\
&= 2
\end{aligned}$$

2. Solve for x.

$$10^{2x+5} = 100^{x^2}$$

To compare the left- and right-hand sides, first let's make their bases equal. Note $10^2 = 100$.

$$10^{2x+5} = 100^{x^2} = (10^2)^{x^2} = 10^{2x^2}$$

Now that the bases are equal, to solve we only need to determine when the exponents are equal. So we solve $2x + 5 = 2x^2$. This is equivalent to solving $2x^2 - 2x - 5 = 0$. Apply the quadratic formula:

$$x = \frac{2 \pm \sqrt{2^2 - (4)(2)(-5)}}{2(2)} = \frac{2 \pm \sqrt{44}}{2(2)} = \frac{2 \pm 2\sqrt{11}}{2(2)} = \frac{1 \pm \sqrt{11}}{2}$$

3. Which expression is equivalent to $4^x 2^5 = 10$?

(a) $\log_4(5) - 2 = x$

(b) $\log_2(10) = 2x + 5$

(c) $x \log(4) + 5 \log(2) = 10$

(d) $\log(10) = 5x \log(8)$

(e) None of the above.

$$
\begin{aligned}
& 4^x 2^5 = 10 \\
\Leftrightarrow\ & (2^2)^x 2^5 = 10 \\
\Leftrightarrow\ & 2^{2x} 2^5 = 10 \\
\Leftrightarrow\ & 2^{2x+5} = 10 \\
\Leftrightarrow\ & \log_2(10) = 2x + 5
\end{aligned}
$$

4. Which expression is equivalent to $y = \log(3^{10} x)$?

(a) $3 + \log x = y$

(b) $y^{10} = 3^{10} x$

(c) $10^y = 3^{10} x$

(d) $10 = y^{3^{10} x}$

(e) None of the above.

$$
\begin{aligned}
& y = \log(3^{10} x) \\
\Leftrightarrow\ & y = \log_{10}(3^{10} x) \\
\Leftrightarrow\ & 10^y = 3^{10} x
\end{aligned}
$$

5. Suppose $f(x)$ is a rational function with domain $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$ and:

- $\lim_{x \to \infty} f(x) = \infty$
- $\lim_{x \to -\infty} f(x) = -\infty$
- $\lim_{x \to -1} f(x)$ exists
- $\lim_{x \to 3^-} = -\infty$
- $\lim_{x \to 3^+} = \infty$
- $f(0) = 0$

(a) What are the horizontal asymptotes of the graph of $y = f(x)$? Express your answer as an equation. If there are no horizontal asymptotes, then write NONE. NONE

If $\lim_{x \to \infty} f(x) = c$ (or $\lim_{x \to -\infty} f(x) = c$) for some number c, then the horizontal asymptote is given by $\lim_{x \to \infty} f(x) = c$. However, since in this case $c = \infty$, there is no horizontal asymptote.

(b) What are the vertical asymptotes of the graph of $y = f(x)$? Express your answer as an equation. If there are no vertical asymptotes, then write NONE. Vertical asymptote: $x = 3$. The candidates for a vertical asymptote are the numbers not in the domain: namely, $x = -1$ and $x = 3$. Since the function doesn't blow up to infinity at $x = -1$, the only vertical asymptote is at $x = 3$.

(c) What are the holes (removable discontinuities) of the graph of $y = f(x)$? Express your answer as an equation. If there are no holes, then write NONE. Hole: $x = -1$. Since the function doesn't blow up to infinity at $x = -1$, but this is excluded from the domain, it must be a hole.

(d) True or false: $f(x)$ is continuous over $(3, 4)$. True: rational functions are continuous over their domains, and $(3, 4)$ is in the domain of $f(x)$.

(e) Suggest a possible rational function for $f(x)$. One answer (of many):

$$f(x) = \frac{x^2(x+1)}{(x+1)(x-3)}$$

Since we exclude $x = -1$ and $x = 3$ from the domain, these values should be the ones that make the denominator of $f(x)$ equal to zero. Then the denominator should include factors $(x + 1)$ and $(x - 3)$.

Since $x = 3$ is a vertical asymptote, the factor $(x - 3)$ shouldn't "cancel". Because the sign changes at $x = 3$, the factor $(x - 3)$ should appear an odd number of times. Since $x = -1$ is a removable discontinuity, the factor $(x + 1)$ should "cancel".

Since $f(0) = 0$, $x = 0$ should make the numerator equal to zero, so x should be a factor of the top. The function goes to infinity as x gets large, so the numerator should have higher degree than the denominator. Since the function goes to negative infinity as x goes to negative infinity, it should have an odd number of terms.