Econ 150 Intermediate Micro Exam 1 Solution

Fall 2008

Question 1.1

We want to solve the following utility maximization problem

$$\max_{x_1,x_2} x_1^2 x_2$$
$$\text{s.t.} \quad x_1 + \frac{x_2}{1+r} = I_1 + \frac{I_2}{1+r}$$

The Lagrange for above maximization problem is

$$\mathcal{L} = x_1^2 x_2 + \lambda(x_1 + \frac{x_2}{1+r} - I_1 - \frac{I_2}{1+r})$$

First order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_1} = 2x_1x_2 + \lambda = 0 \quad (1)$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = x_1^2 + \lambda = 0 \quad (2)$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x_1 + \frac{x_2}{1+r} - I_1 - \frac{I_2}{1+r} = 0 \quad (3)$$

Dividing equation (1) by equation (2) to eliminate the Lagrange multiplier, we get

$$\frac{2x_2}{x_1} = 1 + r$$

which yields

$$x_2 = \frac{(1+r)x_1}{2} \tag{4}$$

Substitute (4) into equation (3) we get

$$x_1 + \frac{(1+r)x_1}{2(1+r)} - I_1 - \frac{I_2}{1+r} = 0 \tag{5}$$

Together with (4) and (5) we get

$$x_1 = \frac{2I_1}{3} + \frac{2I_2}{3(1+r)} \tag{6}$$

$$x_2 = \frac{I_1(1+r)}{3} + \frac{I_2}{3} \tag{7}$$

Question 1.2

When $I_1 = I_2 = 100$ and r = 0.1, we have

$$x_1 = \frac{2}{3} \times 100 + \frac{2}{3} \times \frac{100}{1.1} > I_1$$

Therefore this person is a borrower

Question 1.3

From equation (6) and (7), we see that when r increases, x_1 decreases and x_2 increases. Therefore increase in interest rate will decrease current consumption and increase future consumption. Since $x_1 - I_1$ decreases when x_1 decreases, the amount this person borrows will decrease as r increases.

Question 2.1

$$L = x_1 x_2^3 + \lambda(p_1 x_1 + p_2 x_2 - I)$$

First order conditions:

$$\frac{dL}{dx_1} = x_2^3 + \lambda p_1 = 0$$

$$\frac{dL}{dx_2} = 3x_1 x_2^2 + \lambda p_2 = 0$$

$$\frac{dL}{d\lambda} = p_1 x_1 + p_2 x_2 - I = 0$$

Question 2.2

Bordered Hessian:

$$\begin{bmatrix} L_{11} & L_{12} & L_{1\lambda} \\ L_{21} & L_{22} & L_{2\lambda} \\ L_{\lambda 1} & L_{\lambda 2} & L_{\lambda\lambda} \end{bmatrix} = \begin{bmatrix} 0 & 3x_2^2 & p_1 \\ 3x_2^2 & 6x_1 x_2 & p_2 \\ p_1 & p_2 & 0 \end{bmatrix}$$

The second order conditions are:

$$\begin{vmatrix} 0 & p_1 \\ p_1 & 0 \end{vmatrix} < 0$$

$$\begin{vmatrix} 0 & 3x_2^2 & p_1 \\ 3x_2^2 & 6x_1 x_2 & p_2 \\ p_1 & p_2 & 0 \end{vmatrix} > 0.$$

Question 2.3

A function U is quasiconcave if, for all x, x' and $\lambda \in (0, 1)$, we have

$$U(\lambda x + (1-\lambda)x') \geq \min[U(x), U(x')], \tag{8}$$

An alternate definition is that U is quasiconcave if and only if it has convex upper contour sets. The graph below of the indifference curve for $U(x_1, x_2) = x_1 x_2^3$ shows that it is quasiconcave using the second definition (the gray area is the upper contour set and the labeled point shows that the upper contour set is convex):