Practice #3
Solutions
Waiting Line Management
BUAD311 – Operations Management

1. To support national health week, the American Heart Association plans to install a free blood pressure testing booth in South Coast Plaza for a week. Previous experience indicates that, on the average, 10 persons per hour request a test. Assume the inter-arrival times are exponentially distributed. The blood measurements take a constant time of 4.8 minutes.

   a. What is the utilization of the blood pressure machine?

   $$u = \frac{p}{a*m} = \frac{4.8}{6} = 0.8$$

   b. How long, on average, do people spend in the system (sum of waiting time + blood pressure checking time)?

   $m = 1$

   $p = 4.8\,\text{min}.$

   $a = \frac{60}{10} = 6\,\text{min}$

   $CV_a^2 = 1$

   $CV_s^2 = 0$

   $u = \frac{p}{a*m} = 0.8$

   $$T_q = \frac{p}{m}\frac{u^{\sqrt{2(m+1)}-1}}{1-u}\frac{CV_a^2 + CV_s^2}{2} = 9.6\,\text{minutes}$$

   T=9.6+4.8=14.4 minutes

   c. What is the average number of people in the system (waiting + service)?

   From Little's rule,

   $$I = 10*\left(\frac{14.4}{60}\right) = 2.4 \text{ customers}$$

   d. What is the average waiting time (before you see a nurse)?

   9.6 minutes

   e. What is the average number of people waiting in line?

   10*(9.6/60)=1.6 customers

2. At the California border inspection station, vehicles arrive at the rate of 10 per minute. The inter-arrival times are exponentially distributed. For simplicity assume the there is only one lane and one inspector, who can inspect at the rate of 12 per minute in an exponentially distributed fashion.

a. What is the average waiting time (before inspection)?

$m = 1$

$p = 60/12 = 5$ seconds

a = 60/10 = 6 seconds

$CV_a = CV_s = 1$

$$T_q = \frac{p}{m}\frac{u^{\sqrt{2(m+1)}-1}}{1-u}\frac{CV_a^2 + CV_p^2}{2}$$

$$= \frac{5}{1}\frac{(5/6)}{(1/6)}\frac{1+1}{2}$$

= 25 seconds

b. How long, on average, does a car spend in the system – waiting + inspection?

$T = 25 + 5 = 30$ seconds

c. What is the average number of cars waiting to be inspected?

$I_q = 10 * \frac{25}{60} = 4.17$ vehicles

d. What is the average number of cars in the system – either waiting or being inspected?

$I = 10 * \frac{30}{60} = 5$ vehicles

3. Since the deregulation of the airline industry, fierce competition has forced Global Airlines to reexamine their operations for efficiency and economy. As part of their campaign to improve customer service, they have focused on passenger check-in operations at the terminal. Global operates a common check-in system wherein passengers for all Global's flights line up in a single "snake line". The arrival rate is estimated to be 52 passengers per hour. The check-in process takes an average of three minutes. Agents are paid $20 an hour. The customer relations department estimates that every minute a customer spends waiting in line to be checked in, it costs Global $1 due to missed flights, customers' dissatisfaction, and loss of future business. Assume that inter-arrival and service times are exponentially distributed. In answering the following question you will need the M-M-S Spread Sheet.

a. How many agents should Global staff?

From the SS you can see that with 3 servers the average waiting time is 5.7 minutes. Thus, the total cost is:

3*20 + 5.7*52 = 356

With 4 servers the average waiting time is 0.76 minutes. Thus, the total cost is:

4*20 + 0.76*52 = 119

With 5 servers the average waiting time is 0.19 minutes. Thus, the total cost is:

5*20 + 0.19*52 = 110

With 6 servers the average waiting time is 0.049 minutes. Thus, the total cost is:

6*20 + 0.049*52 =123

Comparing the costs, we find out that 5 servers minimize the cost.

b. What is the average waiting time in line?

0.19 minutes

c. What is the average number of customers in the system?

The average time in the system is: 0.19 + 3 = 3.19min

3.19*(52/60) = 2.76

d. What is the average number of customers that are in service?

Utilization = 52/(5*20) = 52/100

Ip=5*52/100 = 260/100 = 2.6