Practice #6
Solutions
Linear Programming Sensitivity Analysis
BUAD311 – Operations Management

1. Consider the following LP:

Max $3x_1 + 4x_2 + 6x_3 + 10x_4$
Subject to:
$x_1 + x_2 + 3x_4 \leq 120$
$x_1 + x_2 + 3x_3 + x_4 \leq 150$
$x_1 + 2x_2 + x_3 + 2x_4 \leq 100$
$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$

After solving the problem using Excel solver, the following sensitivity repost was generated:

Microsoft Excel 10.0 Sensitivity Report
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
A1	X1	0	-2.2	3	2.2	1E+30
B1	X2	0	-6	4	6	1E+30
C1	X3	40	0	6	24	1
D1	X4	30	0	10	2	5.499999999

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E3		90	0	120	1E+30	30
E4		150	0.4	150	150	50
E5		100	4.8	100	16.66666667	50

(a) What is the value of optimal objective function?

6 * 40 + 10 * 30 = 540

(b) Suppose the first constraint instead was: $x_1 + x_2 + 3x_4$ □ 100. What is the new optimal objective function value? Explain.

540.

The change (-20) is less than the allowable decrease and the shadow price is 0.

(c) Suppose the second constraint instead was: $x_1 + x_2 + 3x_3 + x_4$ □ 190. What is the new optimal objective function value? Explain.

The increase is 40 and is less than the allowable increase (150). Therefore, we can use the shadow price 0.4 to calculate the new objective value. It will be

540 + 40 * 0.4 = 556

(d) Suppose the coefficient multiplying x_3 in the objective function was 10 instead of 6. What is the new optimal objective function value? Why?

The increase is 4 and is less than the allowable increase (24). The optimal decision variables stay the same. The new objective's value is 10*40+10*30=700

(e) Suppose the coefficient multiplying x_4 in the objective function was 13 instead of 10. What is the new optimal objective function value? Why?

The increase is 3 and is more than the allowable increase (2). The optimal decision variables will change. We need to rerun solver to find the new objective's value.

2. Consider the following LP

minimize $2x_1 + 5x_2 + 6x_3 + 12x_4$

subject to:

$x_1 + x_2 + 3x_3 \geq 50$

$x_1 + x_2 + x_3 + 6x_4 \geq 155$

$x_1 + x_2 + 8x_4 \geq 100$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$

Using Excel solver, we have the following sensitivity report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
G5	x1 Objective	50	0	2	1.333333333	0
G6	x2 Objective	0	3	5	1E+30	3
G7	x3 Objective	0	4	6	1E+30	4
G8	x4 Objective	17.5	0	12	0	12

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
J9	$x_1 + x_2 + 3x_3 \geq 50$	50	0	50	105	50
K9	$x_1 + x_2 + x_3 + 6x_4 \geq 155$	155	2	155	1E+30	30
L9	$x_1 + x_2 + 8x_4 \geq 100$	190	0	100	90	1E+30

Note that the optimal objective function value is: $2\times50 + 12\times17.5 = 310$

a) Suppose the third constraint instead was: $x_1 + x_2 + 8x_4 \geq 150$. What is the new optimal objective value? Do you need to re-solve the LP? Explain.

310.
No, we do not need to re-solve as the shadow price of this constraint is 0, and 50 (=150-100) is within the allowable increase.

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b) Suppose the second constraint was: $x_1 + x_2 + x_3 + 6x_4 \geq 130$. What is the new optimal objective value? Do you need to re-solve the LP? Explain.

310 – 2 *(155-130) = 260.
No, we don’t need to re-solve. 25 (=155-130) is less than allowable decrease of 30.

c) Suppose the coefficient multiplying x_4 in the objective function was 2 instead. What is the new optimal objective value? Do you need to re-solve the LP? Explain.