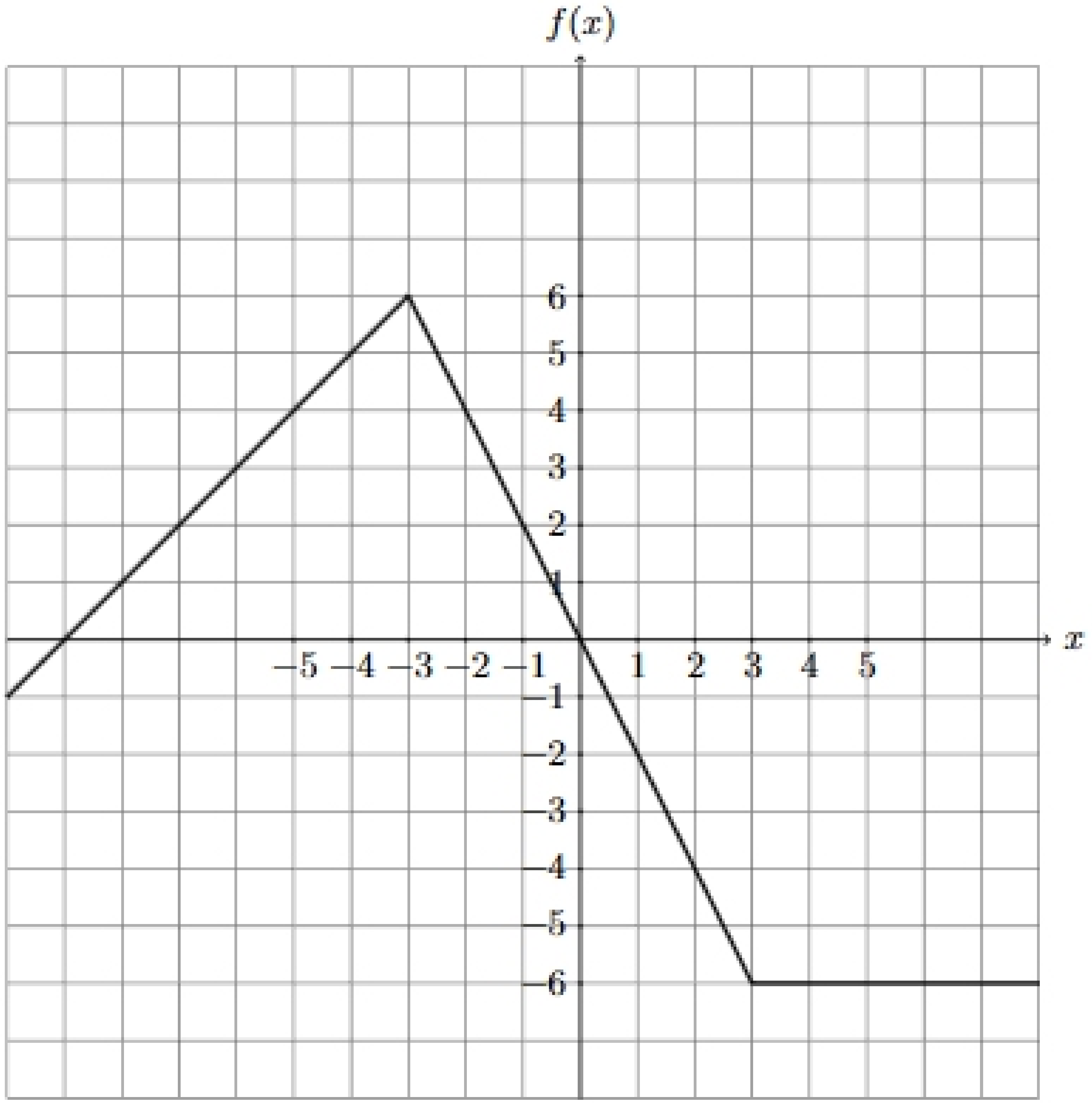
Problem 1. Find the domain and sketch the graph of the function: **2.5-points**

$$f(x) = \begin{cases} x+9 & \text{if } x < -3 \\ -2x & \text{if } x \in [-3, 3] \\ -6 & \text{if } x > 3 \end{cases}$$

Solution: Clearly the given $f(x)$ is piece-wise continuous function, the whole domain(which is $\mathcal{R}$) is divided into three pieces, we will graph these three pieces by considering the respective definition of the function. For the domain $(-\infty, -3)$, let us use $f(x) = x + 9$, which is a equation of line with slope $+1$ (slope is the $x-$coefficient in the given equation of $f(x)$) and when is x is very close to -3, $f(x)$ will be close to 6(**remember we can't take x exactly equal to -3, why?**), then we consider domain $[-3, 3]$, in this domain $f(x) = -2x$, which is the equation of the line with slope -2 and passes through origin(**why?**), finally for the domain $x > 3$, $f(x) = -6$ is a constant function. The graph of the given function is as follows:



Problem 2. Let $f(x) = 4 + 3x + x^2$, then find the value of $\dfrac{f(1+h) - f(1)}{h}$. **2.5-points**

Solution: The given function is $f(x) = 4 + 3x + x^2$, first let us find $f(1+h)$ and $f(1)$,

$$
\begin{aligned}
f(x) &= 4 + 3x + x^2 \\
f(1+h) &= 4 + 3(1+h) + (1+h)^2 \\
f(1+h) &= 4 + 3 + 3h + 1 + 2h + h^2 \\
&\boxed{f(1+h) = 8 + 5h + h^2}
\end{aligned}
$$

Then let us find the value of $f(1)$,

$$
\begin{aligned}
f(x) &= 4 + 3x + x^2 \\
f(1) &= 4 + 3(1) + (1)^2 \\
f(1) &= 4 + 3 + 1 \\
&\boxed{f(1) = 8}
\end{aligned}
$$

Finally, using the values of $f(1+h)$ and $f(1)$ we can find the value of $\dfrac{f(1+h) - f(1)}{h}$

$$
\begin{aligned}
\frac{f(1+h) - f(1)}{h} &= \frac{\cancel{8} + 5h + h^2 - \cancel{8}}{h} \\
&= \frac{5h + h^2}{h} \\
&= \frac{h(5+h)}{h} \\
&\boxed{\frac{f(1+h) - f(1)}{h} = (5+h)}
\end{aligned}
$$

Problem 3. The average surface temperature of the earth has been modeled by a linear function $T = 0.02t + 8.5$, where T is temperature in ^{0}C and t represents years since 1900. **2.5-points**

(a) Find the slope and $T-$intercept, what do these quantities represent?

Slope $= 0.02$ (**slope of a line is the $x-$coefficient in the equation for line.**) Slope represents the rate of change of surface temperature.

$T-$intercept $= 8.5$ (**$T-$intercept can be obtained by taking $t = 0$ in the given equation.**)

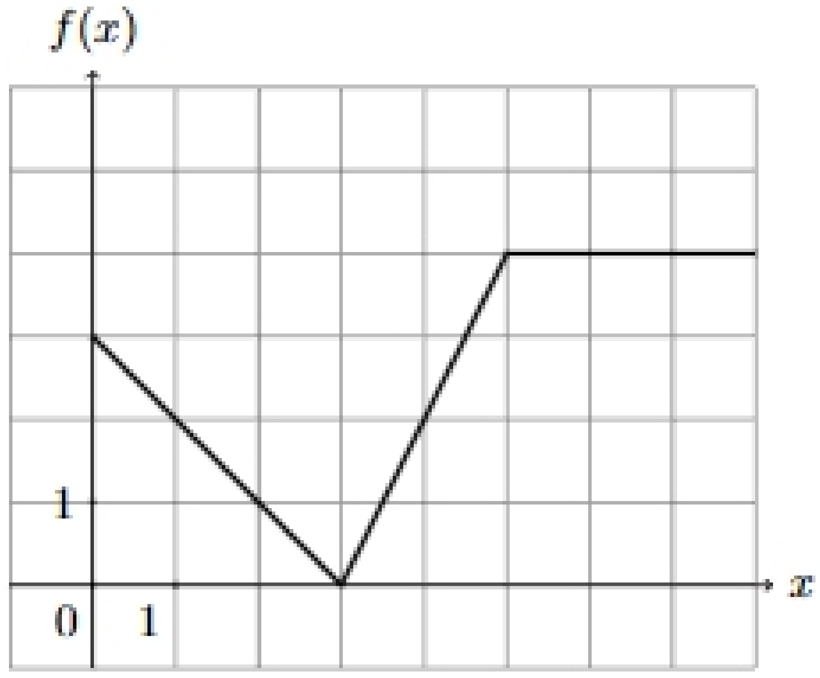
$T-$intercept represents the average surface temperature in the year 1900.

(b) Use the equation to predict the average global surface temperature in 2014?

Recall t represents years since 1900, $\therefore\ t = 2014 - 1900 = 114$, using $t = 114$ in the given model we can predict the average global surface temperature in 2014.

$T(114) = 0.02(114) + 8.5 = 10.78^0$C

Problem 4. Find an expression for the function whose graph is the given curve **2.5-points**



Solution: Clearly, the given function is a piecewise-function with domain $[0, \infty)$, the first piece is defined for $x \in [0, 3]$, the two end points of the line are $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (3, 0)$ **(recall that to find equation of a line we need two points on the line)**, we then calculate the slope of the line

$$\text{Slope, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{3 - 0} = -1.$$

Then slope-point form of line is:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= (-1)(x - 3) \\ \therefore\ y &= -x + 3 \end{aligned}$$

Similarly the equation of the line with the endpoints $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (5, 4)$ is given by:

$$\text{Slope, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{5 - 3} = 2.$$

Then slope-point form of line is:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 2(x - 3) \\ \therefore\ y &= 2x - 6. \end{aligned}$$

Finally, for $x > 5$ $f(x) = 4$ is a constant function.
Combining all the above:

$$f(x) = \begin{cases} -x + 3 & \text{if } x \in [0, 3] \\ 2x - 6 & \text{if } x \in [3, 5] \\ 4 & \text{if } x > 5 \end{cases}$$