
Problem 1. If $f(x) = x^5 + x^3 + x$, then find the value of $f^{-1}(3)$ and $f(f^{-1}(2))$.
Solution: To find $f^{-1}(3)$, we first need to find a such that $f(a) = 3$, by inspection $f(1) = 3$, hence $f^{-1}(3) = 1$.
Since the given function is a *polynomial*, the domain of f is all real numbers, $\therefore$ 2 is in the domain of $f(x)$, hence $f(f^{-1}(2)) = 2$.

Problem 2. Find the domain of f and f^{-1}, given that $f(x) = \ln(2 + \ln x)$
Solution: First let us find the domain of f, since the given function is $\ln(\cdot)$, we want

$$2 + \ln x > 0$$

Let us solve the above inequality for x to get the domain of f

$$\ln_e x > -2$$

convert the above equation to exponential form

$$x > e^{-2}$$

$\therefore$ the domain of f is (e^{-2}, ∞).
Next, to find a formula for f^{-1} we will apply ***two-step procedure***:

$$\begin{aligned}
y &= \ln(2 + \ln x) \quad \textbf{interchange } x \textbf{ and } y \\
x &= \ln(2 + \ln y) \quad \textbf{solve for } y \textbf{ in terms of } x \\
\ln_e(2 + \ln_e y) &= x \\
2 + \ln_e y &= e^x \\
\ln_e y &= e^x - 2 \\
y &= e^{e^x - 2}
\end{aligned}$$

$\therefore \quad f^{-1}(x) = e^{e^{-x}-2}$, the domain of f^{-1}(or the range of f) is all real numbers($\mathbb{R}$).

Problem 3. Solve the equation for x:

$$e^{2x} - 3e^x + 2 = 0.$$

Solution: First we will rewrite the given equation as:

$$(e^x)^2 - 3e^x + 2 = 0.$$

continued $\cdots\cdots\cdots$

Let us denote $e^x = a$ in the above equation,

$$(a)^2 - 3a + 2 = 0,$$

which is a ***Quadratic equation***, to solve this we will factorize first

$$(a-2)(a-1) = 0,$$

the solutions are $a = 1$ and $a = 2$, but we know that $a = e^x$,

$$\begin{aligned} e^x &= a \\ e^x &= 1 \\ \text{taking natural log on both sides} \\ \ln e^x &= \ln 1 \\ x \ln e &= 0 \\ x &= 0. \end{aligned}$$

Similarly for $a = 2$, we get $x = \ln 2$.

Problem 4. Evaluate the limit, if it exists $\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

Solution:

$$\begin{aligned} \lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right) &= \lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t(1+t)}\right) \\ &= \lim_{t\to 0}\left(\frac{t+1}{t(t+1)} - \frac{1}{t(1+t)}\right) \\ &= \lim_{t\to 0}\left(\frac{t+1-1}{t(t+1)}\right) \\ &= \lim_{t\to 0}\left(\frac{t}{t(t+1)}\right) \\ &= \lim_{t\to 0}\left(\frac{1}{(t+1)}\right) \\ &= \frac{1}{0+1} \\ &= 1 \end{aligned}$$