Name: Solution Due: 2-28-14

SPRING-2014 **MATH 131:503** **QUIZ - 4**

- **Read each question carefully, and answer each question completely.**
- **Show all of your work; no credit will be given for unsupported answers.**

Problem 1. Find the numbers at which the function $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ e^x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}$ **2.5-points**

is discontinuous. At which of these points is f continuous from the right, from the left, or neither? Sketch the graph of f.

At $x=0$

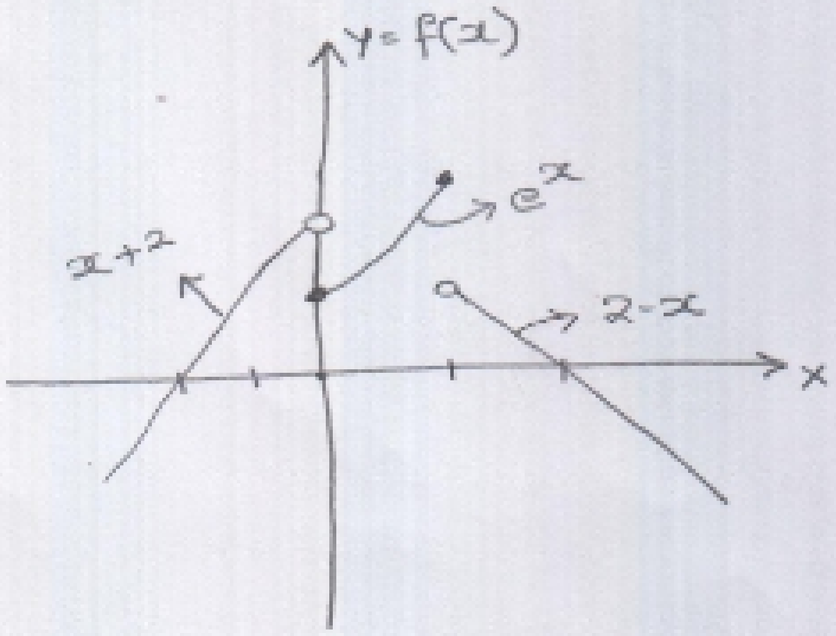
$LHL = \lim_{x\to 0} f(x) = \lim_{x\to 0} x+2 = 0+2 = 2$

$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} e^x = e^0 = 1$

At $x=0$ $f(x) = e^x$

$f(0) = e^0 = 1$

Continuous from right at $x=0$



At $x=1$, $LHL = \lim_{x\to 1^-} f(x) = \lim_{x\to 1} e^x = e^1$

$RHL = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} 2-x = 1$

$f(x) = e^x$ $f(1) = e$, Continuous from left at $x=1$

Problem 2. Find the derivative using the definition of $f(x) = \frac{1}{2}x - \frac{2}{3}$ **2.5-points**

$f(x) = \frac{1}{2}x - \frac{2}{3}$ $f(x+h) = \frac{1}{2}(x+h) - \frac{2}{3}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{2}(x+h) - \frac{2}{3}\right) - \left(\frac{x}{2} - \frac{2}{3}\right)}{h}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{\frac{x}{2}} + \frac{h}{2} - \cancel{\frac{2}{3}} - \cancel{\frac{x}{2}} + \cancel{\frac{2}{3}}}{h}$

$= \lim_{h\to 0} \frac{h}{2} \div h$

$= \lim_{h\to 0} \frac{\cancel{h}}{2} \cdot \frac{1}{\cancel{h}}$

continued ………

$f'(x) = \frac{1}{2}$

Problem 3. Find the value of $\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)$ **2.5-points**

$$\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right) * \frac{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+ax}\right)^2+\cancel{\sqrt{x^2+ax}\sqrt{x^2+bx}}-\cancel{\sqrt{x^2+bx}\sqrt{x^2+ax}}-\left(\sqrt{x^2+bx}\right)^2}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty}\frac{\cancel{x^2}+ax-\cancel{x^2}-bx}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} = \lim_{x\to\infty}\frac{\frac{ax}{x}-\frac{bx}{x}}{\frac{\sqrt{x^2+ax}}{\sqrt{x^2}}+\frac{\sqrt{x^2+bx}}{\sqrt{x^2}}}$$

$$= \lim_{x\to\infty}\frac{a-b}{\sqrt{\frac{x^2}{x^2}+\frac{ax}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{bx}{x^2}}} = \lim_{x\to\infty}\frac{a-b}{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}}$$

$$= \frac{a-b}{\sqrt{1+0}+\sqrt{1+0}} = \frac{a-b}{2}$$

Problem 4. Find the derivative using the definition of $f(x)=\sqrt{x}+x$ **2.5-points**

$$f(x)=\sqrt{x}+x$$

$$f(x+h)=\sqrt{x+h}+x+h$$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{\sqrt{x+h}+\cancel{x}+h-\sqrt{x}-\cancel{x}}{h}$$

$$f'(x)=\lim_{h\to0}\frac{\sqrt{x+h}+h-\sqrt{x}}{h} * \frac{\sqrt{x+h}+h+\sqrt{x}}{\sqrt{x+h}+h+\sqrt{x}}$$

$$=\lim_{h\to0}\frac{(\sqrt{x+h}+h-\sqrt{x})(\sqrt{x+h}+h+\sqrt{x})}{h(\sqrt{x+h}+h+\sqrt{x})}$$

$$=\lim_{h\to0}\frac{\cancel{x}+h+h\sqrt{x+h}+\cancel{\sqrt{x}\sqrt{x+h}}+h\sqrt{x+h}+h^2+\cancel{h\sqrt{x}}-\cancel{\sqrt{x}\sqrt{x+h}}-\cancel{h\sqrt{x}}-\cancel{x}}{h(\sqrt{x+h}+h+\sqrt{x})}$$

$$f'(x)=\lim_{h\to0}\frac{\cancel{h}(1+2\sqrt{x+h}+h)}{\cancel{h}(\sqrt{x+h}+h+\sqrt{x})}$$

take $h=0$

$$f'(x)=\frac{1+2\sqrt{x}}{\sqrt{x}+\sqrt{x}}$$

$$=\frac{1+2\sqrt{x}}{2\sqrt{x}}$$

$$f'(x)=\frac{1}{2\sqrt{x}}+\frac{2\sqrt{x}}{2\sqrt{x}}$$

$$\boxed{f'(x)=\frac{1}{2\sqrt{x}}+1}$$