HW # 3

Problem 1 $L(t) = 10 + 0.01t$ cm $\quad L(0) = 10$ cm

Assuming a homogeneous deformation, the stretch ratio $\Lambda = \frac{L(t)}{L(0)} = \frac{10 + 0.01t}{10}$

One dimensional motion for a homogeneous deformation

$\vec{u} : x = \Lambda X$, the displacement is $\vec{u} = u_x \hat{i}$,

$u_x = x - X = (\Lambda - 1)X$

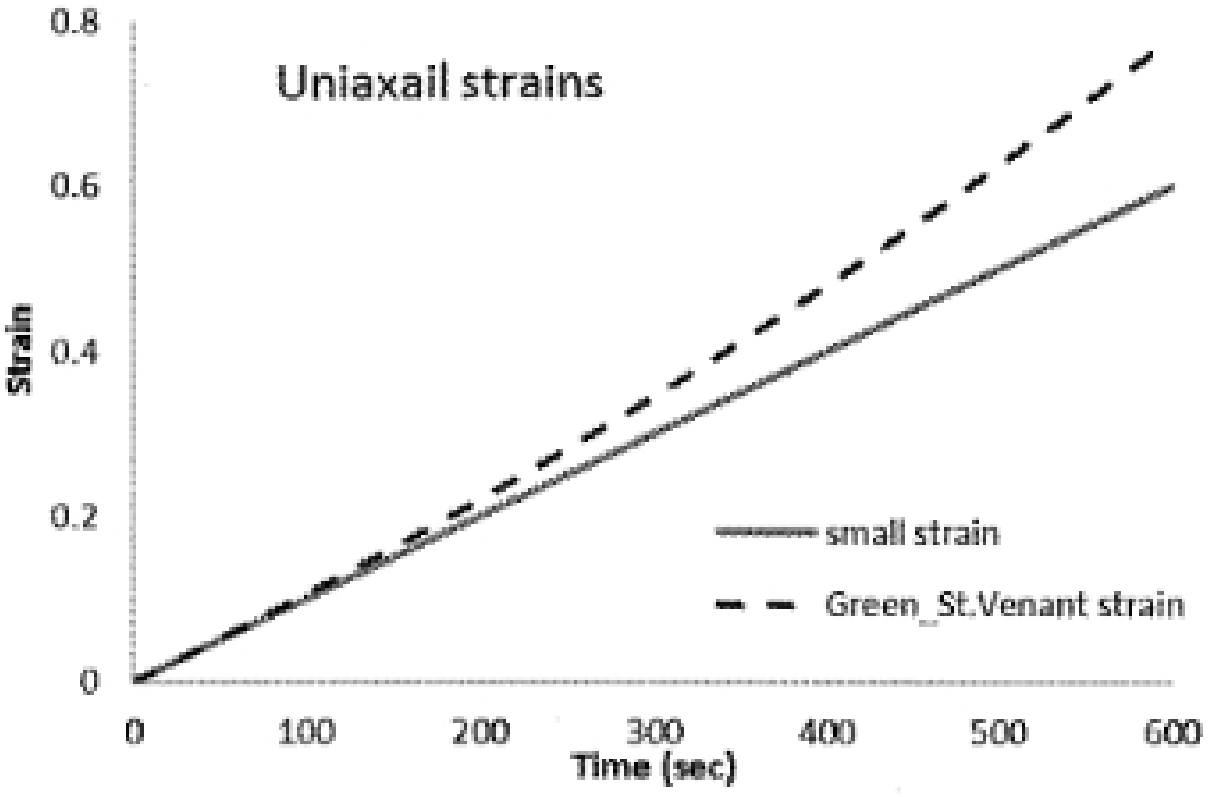
$$\frac{\partial u_x}{\partial X} = (\Lambda - 1) = \frac{10 + 0.01t}{10} - 1 = \frac{0.01t}{10}$$

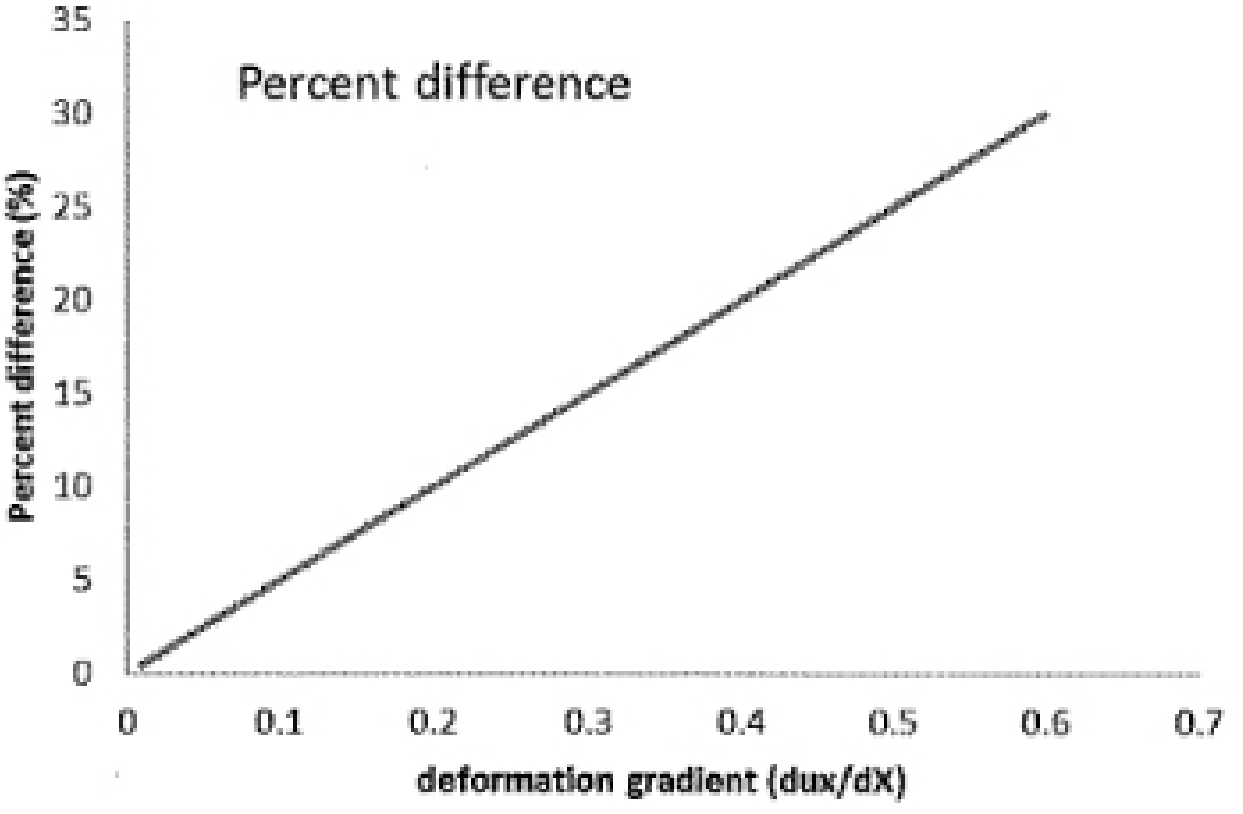
The small strain is $E_{xx} = \frac{\partial u_x}{\partial X} = \frac{0.01t}{10}$

the Green-Lagrange strain is $E_{xx} = \frac{1}{2}\left[\frac{\partial u_x}{\partial X} + \frac{\partial u_x}{\partial X} + \left(\frac{\partial u_x}{\partial X}\right)^2\right]$

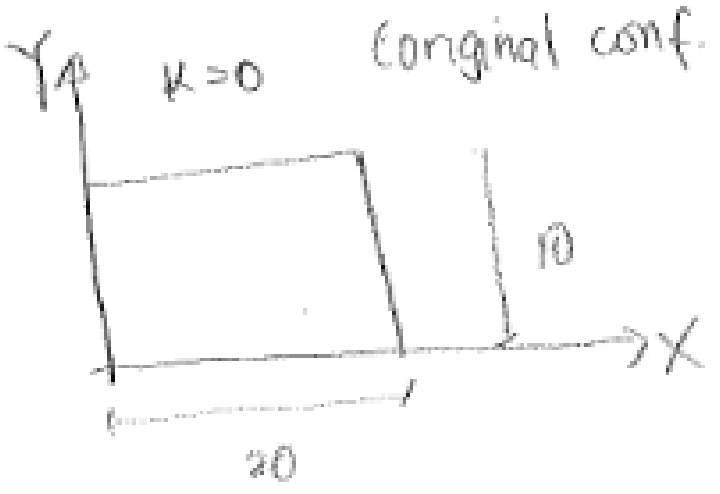
$$= \frac{0.01t}{10} + \frac{1}{2}\left(\frac{0.01t}{10}\right)^2$$

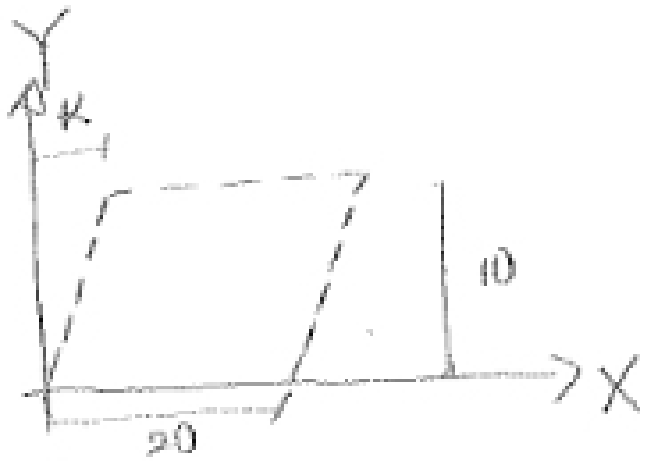
Name:


Uniaxail strains
Strain
0.8
0.6
0.4
0.2
0
small strain
Green_St.Venant strain
0
100
200
300
400
500
600
Time (sec)


Percent difference
Percent difference (%)
35
30
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
deformation gradient (dux/dX)

Problem 2





$x = \frac{K}{10} Y + X$

$y = Y$

the dashed line is constructed using these two equations with $0 \le X \le 20$ and $0 \le Y \le 10$

The displacement vector $\vec{u} = u_x \hat{i} + u_y \hat{j}$

$u_x = x - X = \frac{K}{10} Y$ $\qquad u_y = Y - Y = 0$

$\frac{\partial u_x}{\partial X} = 0$ $\qquad \frac{\partial u_x}{\partial Y} = \frac{K}{10}$ $\qquad \frac{\partial u_y}{\partial X} = 0$ $\qquad \frac{\partial u_y}{\partial Y} = 0$

Small strains:

$\epsilon_{xx} = 0$ $\qquad \epsilon_{yy} = 0$ $\qquad \epsilon_{xy} = \frac{1}{2} \frac{K}{10}$

Green-St. Venant strains

$E_{xx} = 0$ $\qquad E_{yy} = \frac{1}{2} \left[\frac{\partial u_x}{\partial Y} \right]^2 = \frac{1}{2} \frac{K^2}{100}$ $\qquad E_{xy} = \frac{1}{2} \left[\frac{\partial u_x}{\partial Y} \right] = \frac{1}{2} \frac{K}{10}$

$K = 0.01$ $\qquad E_{yy} = 5 \times 10^{-7} \approx 0$ $\qquad \epsilon_{xy} = E_{xy} = 5 \times 10^{-4}$

$K = 1$ $\qquad E_{yy} = 5 \times 10^{-3} \neq 0$ $\qquad \epsilon_{xy} = E_{xy} = 5 \times 10^{-2}$

$K = 5$ $\qquad E_{yy} = 0.125 \neq 0$ $\qquad \epsilon_{xy} = E_{xy} = 0.25$