CS570

Analysis of Algorithms

Fall 2014

Exam I

Name: ________________________

Student ID: ____________________

Thursday Evening Section **DEN Yes / No**

	Maximum	Received
Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	20	
Problem 5	15	
Problem 6	15	
Total	100	

2 hr exam
Close book and notes

If a description to an algorithm is required please limit your description to within 150 words, anything beyond 150 words will not be considered.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]

Adding a number *w* on the weight of every edge of a graph might change the shortest path between two vertices u and v.

True:

Consider G = (V, E) with V = {u, x, v} and E = {ux, xv, uv}. Let weight of edge ux = w_{ux} = w_{xv} = 3 and w_{uv} = 7. Then the shortest path from u to v is given by u->x->v with a total weight of 6. However, if we now add 2 to every edge weight, the path u->x->v will have a total weight of 10 while the path u->v will have a weight of 9, making it the new shortest path.

[TRUE/FALSE]

Suppose that for some graph G we have that the average edge weight is A. Then a minimum spanning tree of G will have weight at most $(n-1) \cdot A$.

False:

We may be forced to select edges with weight much higher than average. For example, consider a graph G consisting of a complete graph G' on 4 nodes, with all edges having weight 1 and another vertex u, connected to one of the vertices of G' by an edge of weight 8. The average weight is (8 + 6)/7 = 2. Therefore, we would expect the spanning tree to have weight at most 4*2 = 8. But the spanning tree has weight more than 8 because the unique edge incident on u must be selected.

[TRUE/FALSE]

DFS finds the longest paths from start vertex *s* to each vertex *v* in the graph.

False:

Depends on the order in which the nodes are traversed

[TRUE/FALSE]

If one can reach every vertex from a start vertex s in a directed graph, then the graph is strongly connected.

False

[TRUE/FALSE]

$F(n) = 4n + 3\sqrt{n}$ is both $O(n)$ and $\Omega(n)$.

True:

The dominant term is 4n, which is obviously both O(n) and \Omega (n)

[TRUE/FALSE]
In Fibonacci heaps, the decrease-key operation takes $O(1)$ time.

True:
Just as definition of decrease-key operation in Fibonacci heaps

[TRUE/FALSE]
If the edge weights of a weighted graph are doubled, then the number of minimum spanning trees of the graph remains unchanged.

True:
With edge weights doubled, the weights of all possible spanning trees in the graph are doubled. So any MST in the original graph is also the MST in the new graph; any spanning tree that is not the MST in the original graph is still not the MST in the new graph.

[TRUE/FALSE]
Given a binary max-heap with n elements, the time complexity of finding the smallest element is O(lg n).

False:
The smallest element should be among the leaf nodes. Consider a full binary tree of n nodes. It has (n+1)/2 leafs (you can think of why). Then the worst case of finding the smallest element of a full binary tree (heap) is \Theta(n)

[TRUE/FALSE]
An undirected graph $G = (V, E)$ must be connected if $|E| > |V| - 1$

False:
Consider a graph having nodes: a, b, c, d, e. {a, b, c, d} forms a fully connected subgraph, while e is isolated from other nodes. Now the fully connected subgraph has 6 edges, and there are only 5 nodes in total. But this graph is not a connected graph.

[TRUE/FALSE]
If all edges in a connected undirected graph have unit cost, then you can find the MST using BFS.

True:
Any spanning tree of a graph having only unit cost edges is also a MST, because the weight is always n-1 units. Of course, BFS gives a spanning tree in the connected graph.