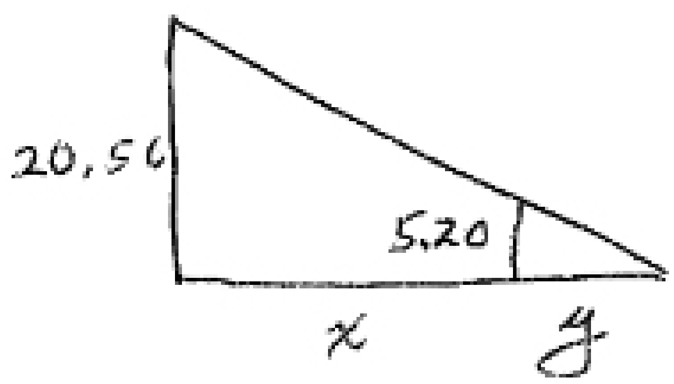
Extra Exam #1, Name: SOLUTIONS
MATH 132, FALL 2004 FIRST EXAMINATION, FIRST CHANCE KEY = 127

1. A woman 5.20 feet tall walks at a rate of 6.02 feet per second away from a street light that is 20.56 feet above the street. At what rate is the length of her shadow changing, in feet per second?



By similarity of the large and small triangles:

$$\frac{x+y}{y} = \frac{20.56}{5.20} = 3.9538$$

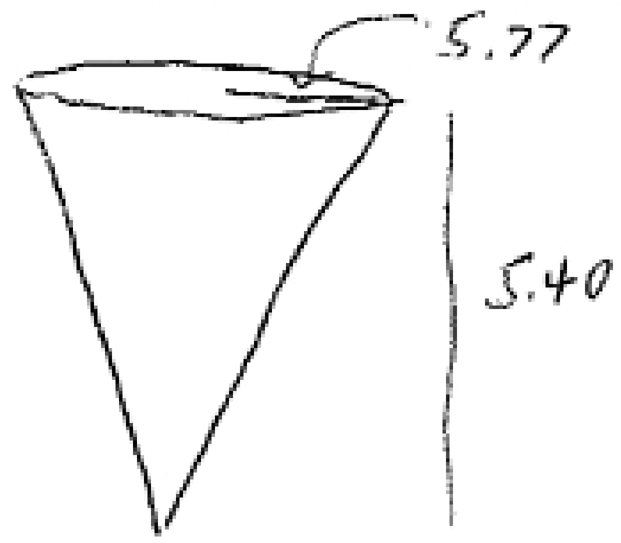
$$\frac{x}{y} = 2.9538, \quad x = 2.9538\, y,$$

$$y = 0.33854\, x$$

$$\frac{dy}{dt} = 0.33854 \frac{dx}{dt}$$

$$= 0.33854 \times 6.02 = 2.038$$

2. Water is being withdrawn from a conical reservoir (with point down) 5.77 feet in radius and 5.40 feet deep at 8.23 cubic feet per minute. How fast is the surface falling when the depth of the water is 4.37 feet?



(not to scale, obviously)

$$V = \frac{1}{3}\pi r^2 h \quad \text{and } r = \frac{5.77}{5.40} h,$$

$$\text{so } V = \frac{1}{3}\pi \left(\frac{5.77}{5.40}\right)^2 h^3 = 1.1956\, h^3$$

$$\frac{dV}{dt} = 3.5869\, h^2 \frac{dh}{dt}$$

$$8.23 = 3.5869 \times 4.37^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8.23}{3.5869 \times 4.37^2} = 0.1201$$

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3. The radius of a ball of ice shrinks from 8.24 feet to 6.69 feet. Use the method of differentials to estimate the decrease in volume, in cubic feet.

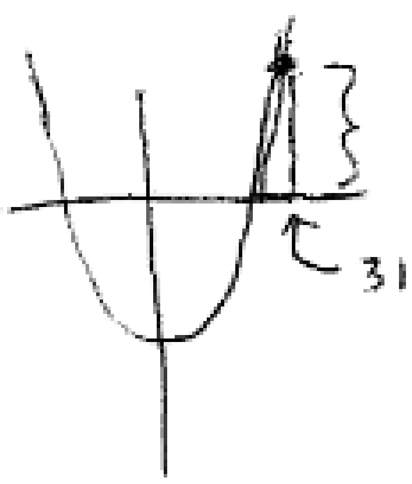
$\Delta r = 6.69 - 8.24 = -1.55$

$V = \frac{4}{3}\pi r^3, \quad dV = 4\pi r^2 \Delta r$

$\Delta V \doteq dV = 4\pi\, 8.24^2 (-1.55) = -1322.5$

Since the problem asked for the decrease, we can report the value without the minus: 1322.5

4. Using Newton's method to find the square root of x = 133.1 by solving the equation x² - 133.1 = 0 beginning at x = 31, give the value of x that you obtain in the first step.



$31^2 - 133.1 = 827.5 \qquad \frac{dy}{dx} = 2x = 62,$

$x = 31 - 827.5/62 = 17.647$

(not to scale)

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5. Find:

$$\int_{0.33}^{0.50} 1 / (1 + \cos(0.33x)) \quad dx$$

(Note: This problem comes from the chapter on antidifferentiation, but it uses randomly chosen limits in order to make it have a numeric answer.)

$$\int \frac{1}{1+\cos ax}\,dx = \int \frac{1-\cos ax}{\sin^2 ax}\,dx = \int \csc^2 ax\,dx - \int \frac{\cos ax}{\sin^2 ax}\,dx$$

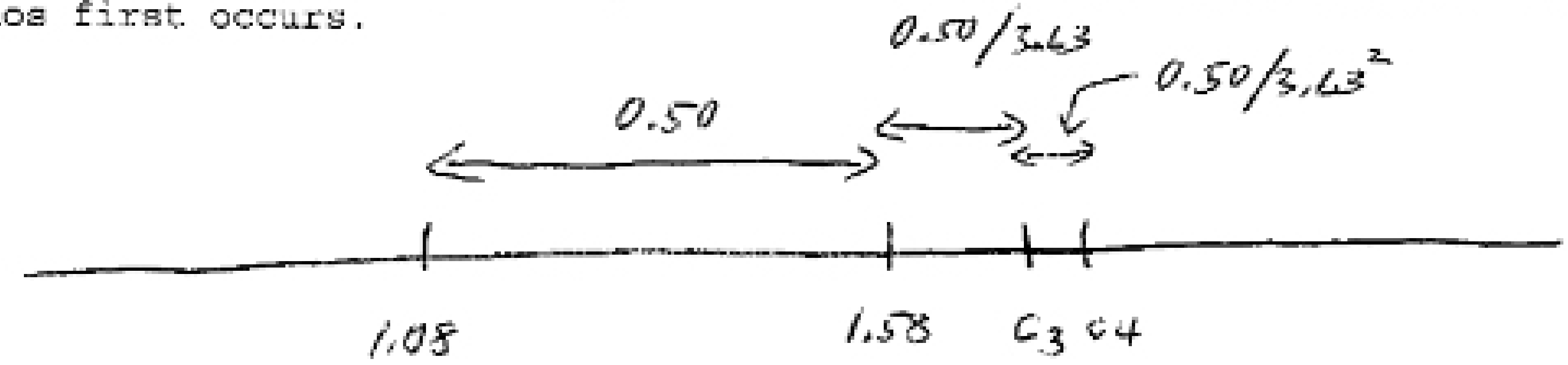
$$= \frac{1}{a}\left(-\cot ax + \frac{1}{\sin ax}\right) = \frac{1}{a}\left(\frac{1}{\sin ax} - \frac{1}{\tan ax}\right)$$

Now let $a = 0.33$ and evaluate at the limits:

$$\frac{1}{0.33}\left(\frac{1}{\sin 0.33x} - \frac{1}{\tan 0.33x}\right)\Bigg]_{0.33}^{0.50} =$$

$$\frac{1}{0.33}\left(6.08819 - 6.00551 - 9.20091 + 9.14641\right) = 0.0854$$

6. Suppose in a system that eventually becomes chaotic after infinitely many period doublings, that the first period doubling occurs at C1 = 1.08 and the second period doubling occurs at C2 = 1.58. Suppose that all ratios (Cn - Cn-1)/(Cn+1 - Cn) are equal to 3.63. Determine value of C for which chaos first occurs.



The successive distances form a geometric series with starting term $a = 0.50$ and ratio $r = \frac{1}{3.63} = 0.27548$. Therefore the total distance from C_1 to C is $\frac{0.5}{1-0.27548} = 0.69001$ and the value of $C = 0.69001 + C_1$

$= 1.770$