Math 216 Fall 2015 midterm 2

Name: ______________________________

Uniqname: ______________________________

UMID: ______________________________

Check one:						
Section	010	020	030	040	050	060
Class hours	9a–10a	10a–11a	11a–12p	12p–1p	1p–2p	2p–3p
Instructor	Colombo		Goluskin		Elling	

1. Turn off your phone and other noisemakers.
2. You may use only pens/pencils and exam paper. No calculators, smartphones or other electronic devices; no books, notes, cards, cheatsheets. All those must remain in your closed bag.
3. This exam has 8 problems and 10 pages. If there is a page missing, notify a proctor.
4. Do not separate the pages. If the exam pages get separated, write your name on *every* page and at the end ask the proctors to staple them.
5. Please read the questions *carefully*. The proctors will not answer questions during the exam.
6. Provide sufficient explanation for each answer.
7. You have 110 minutes to complete the exam.
8. You must remain in the room for at least 70 minutes.
9. If you finish early, use your time to check your answers.
10. If you get stuck on a problem, try the next one and come back later.
11. Some integrals that may or may not be useful:

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \frac{1}{x} \, dx = \log|x| + C \qquad \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$

$$\int \frac{1}{1+x^2} \, dx = \arctan x + C \qquad \int \frac{1}{1-x^2} \, dx = \operatorname{arctanh} x + C$$

Problem 1

$$\mathbf{y}'(t) = \mathbf{f}(t, \mathbf{y}(t)) \quad , \quad \mathbf{f}(t, \mathbf{y}) = t\mathbf{y} \quad , \quad \mathbf{y}(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Perform one step of the Euler method to estimate $\mathbf{y}(1.1)$.

Solution $h = 0.1$, $t_0 = 1$,

$$y_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and

$$y_1 = y_0 + hf(t_0, y_0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0.1 \cdot 1 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 2.2 \end{bmatrix}$$

Problem 2

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.
(b) For each eigenvalue, give a corresponding eigenvector.

Solution (a) Characteristic equation:

$$0 \stackrel{!}{=} \det(A - \lambda I) = (1-\lambda)(2-\lambda) - 1 \cdot 0$$

The roots are $\lambda_1 = 1$ and $\lambda_2 = 2$ (for triangular matrices the diagonal elements are the eigenvalues). (b) For $\lambda_1 = 1$:

$$0 \stackrel{!}{=} (A - \lambda_1 I)v_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \begin{bmatrix} v_{21} \\ v_{21} \end{bmatrix}$$

Hence $v_{21} = 0$ while v_{11} is arbitrary. For example

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

or any nonzero multiple thereof. For $\lambda_2 = 2$, we obtain in similar fashion

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

or any nonzero multiple thereof.