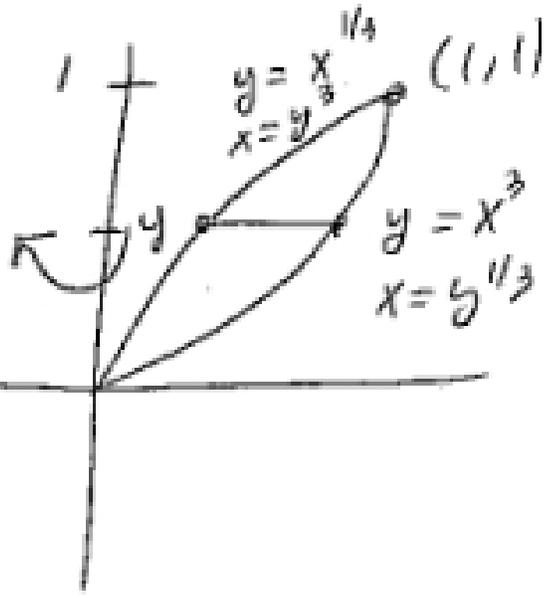
MATH 132 **EXAM II** FALL 2009

This exam contains 16 multiple choice questions and 2 hand graded questions. The multiple choice questions are worth 5 points each and the hand graded questions are worth a total of 20 points. The latter questions will be evaluated not only for having the correct solutions but also for **clarity**. Points may be taken off for confusing and disorganized writing, even when the answer is correct.

1) Find the volume of the solid obtained by rotating the region in the first quadrant enclosed by the graphs $y = x^3$, $y = x^{\frac{1}{3}}$, about the y-axis.

A) $\frac{3\pi}{10}$
B) $\frac{5\pi}{2}$
C) $\frac{32\pi}{3}$
D) $\frac{9\pi}{8}$
E) $\frac{2\pi}{5}$
F) $\frac{16\pi}{35}$
G) $\frac{7\pi}{16}$
H) $\frac{5\pi}{24}$
I) $\frac{3\pi}{32}$
J) $\frac{20\pi}{35}$



Disc:

$$\pi \int_0^1 (y^{2/3} - y^6)\, dy$$

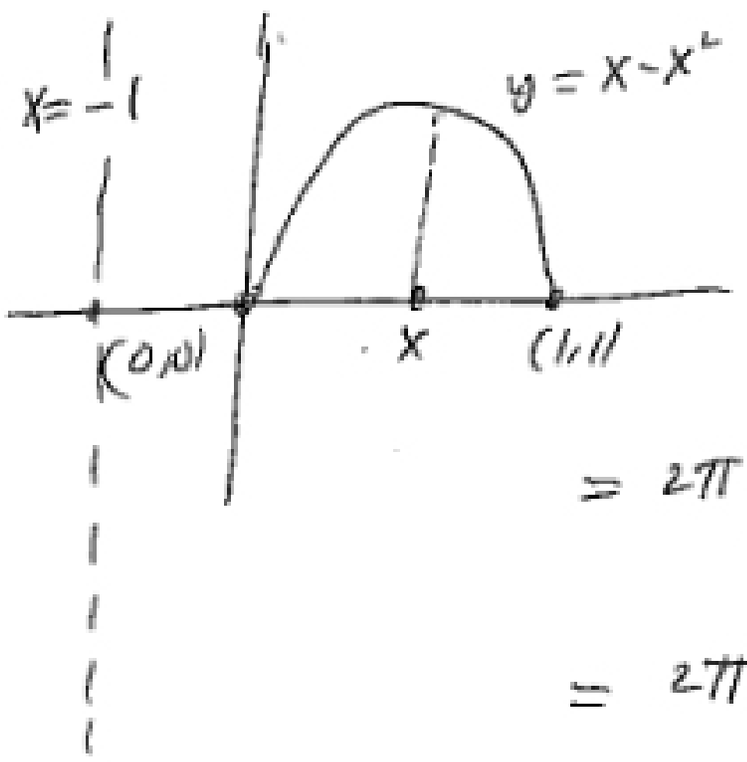
$$= \pi \left(\frac{3}{5} y^{5/3} - \frac{1}{7} y^7 \right) \Big|_0^1$$

$$= \pi \left(\frac{3}{5} - \frac{1}{7} \right) = \frac{16\pi}{35}$$

F

2) Find the volume of the solid obtained by rotating the region enclosed by the parabola $y = x - x^2$ and the x-axis about the vertical line $x = -1$.

A) $\frac{3\pi}{4}$
B) $\frac{4\pi}{3}$
C) $\frac{\pi}{3}$
D) $\frac{\pi}{2}$
E) $\frac{7\pi}{5}$
F) $\frac{2\pi}{3}$
G) $\frac{3\pi}{2}$
H) $\frac{5\pi}{4}$
I) $\frac{3\pi}{5}$
J) $\frac{5\pi}{3}$



$x - x^2 = 0 \Rightarrow x(1-x) = 0$

$x = 0, x = 1$

Shell Method:

$$2\pi \int_0^1 (x+1)(x - x^2)\, dx$$

$$= 2\pi \int_0^1 \cancel{x^2} + x - x^3 \cancel{- x^2}\, dx$$

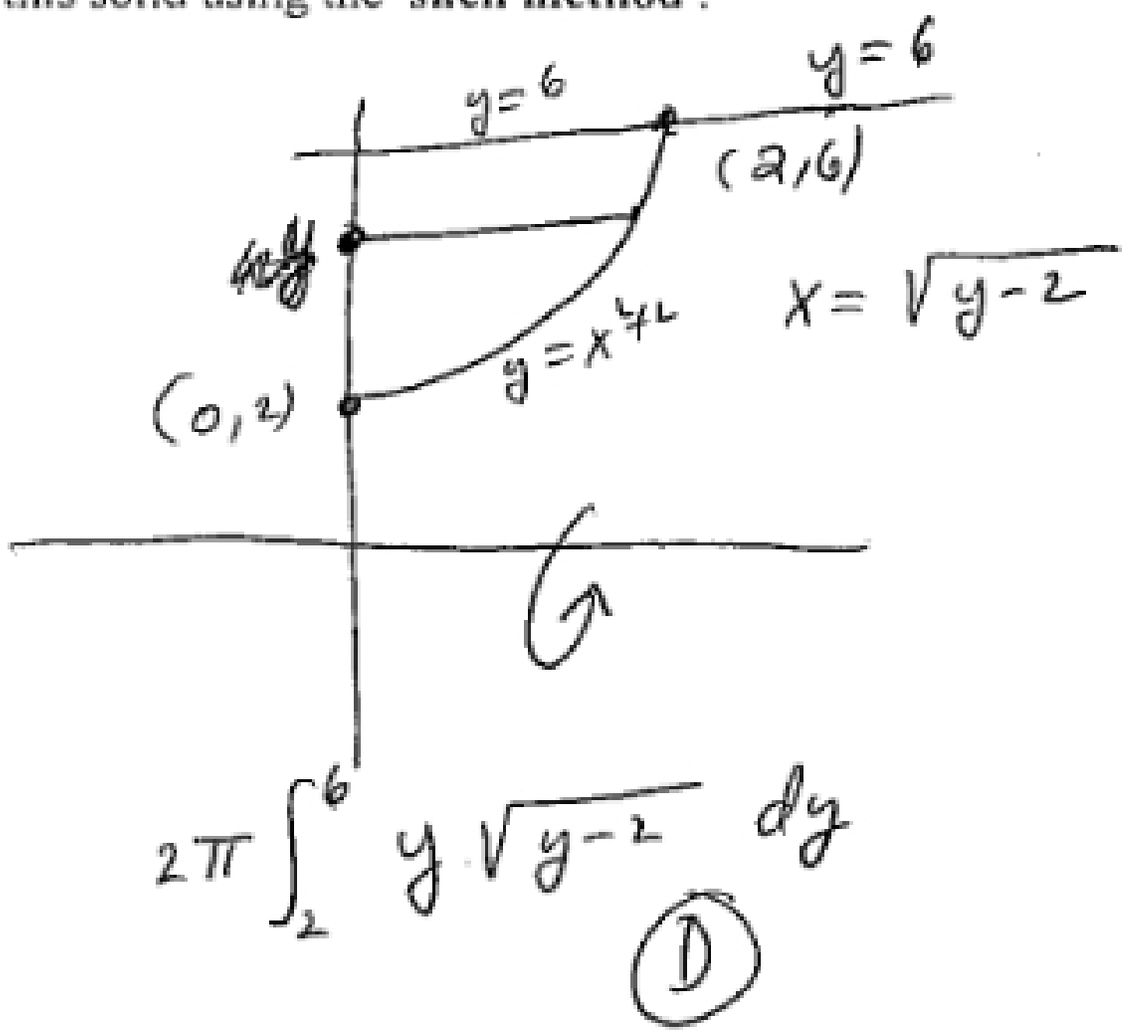
$$= 2\pi \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = 2\pi \left(\frac{1}{2} - \frac{1}{4} \right)$$

$$= \frac{\pi}{2}$$

D

3) Consider the solid obtained by rotating about the **x-axis** the region in the first quadrant enclosed by $y = x^2 + 2$, $y = 6$, and $x = 0$. Write out the integral you would use to find the **volume** of this solid using the **shell method** .

A) $\pi \int_0^2 (x^2 + 2)^2 \, dx$
B) $2\pi \int_0^2 x(x^2 + 2) \, dx$
C) $\pi \int_0^6 y\sqrt{y-2} \, dy$
D) $2\pi \int_2^6 y\sqrt{y-2} \, dy$
E) $\pi \int_2^6 (x^2 + 2)^2 \, dx$
F) $2\pi \int_0^2 x(x^2 + 2) \, dx$
G) $\pi \int_2^6 \sqrt{y-2} \, dy$
H) $2\pi \int_0^6 y\sqrt{y-2} \, dy$
I) $\pi \int_0^6 x(x^2 + 2) \, dx$
J) $2\pi \int_0^6 \sqrt{y-2} \, dy$

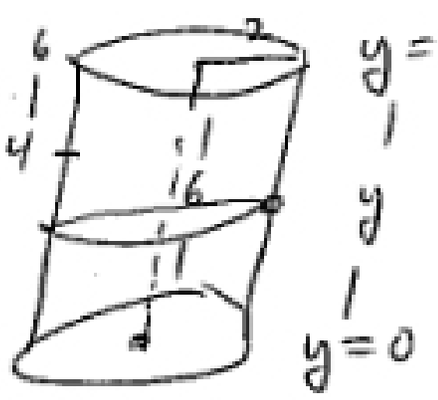


$$2\pi \int_2^6 y\sqrt{y-2} \, dy$$

(D)

4) A tank in the shape of a **cylinder** with radius 2 ft and height 6 ft is **full** of a liquid weighing 80 lb/ft^3. Find the **work** in ft-lb needed to pump **one-third** of the liquid out of the **top** of the tank.

A) 120 π
B) 230 π
C) 345 π
D) 460 π
E) 520 π
F) 640 π
G) 730 π
H) 840 π
I) 890 π
J) 940 π



$A(y) = 4\pi$

$V(y) = 4\pi \, \Delta y$

$F(y) = 320\pi \, \Delta y$

$W(y) = 320\pi (6-y) \, \Delta y$

$$W = \int_4^6 320\pi (6-y) \, dy$$

$$= 320\pi \left[6y - \frac{y^2}{2} \right]_4^6 =$$

$$= 320\pi \left[(36 - 18) - (24 - 8) \right]$$

$$= 640\pi$$

5) It takes 1800 ft-lb of **work** to stretch a spring 3 ft from its **natural length.** How many feet beyond its natural length will a **force** of 2000 lb **stretch the spring ?**

A) 1
B) 2
C) 3
D) 4
E) 5
F) 6
G) 7
H) 8
I) 9
J) 10

natural

$x = 0$

Solve

$2000 = 400\ x$

$\Rightarrow x = 5$

Given

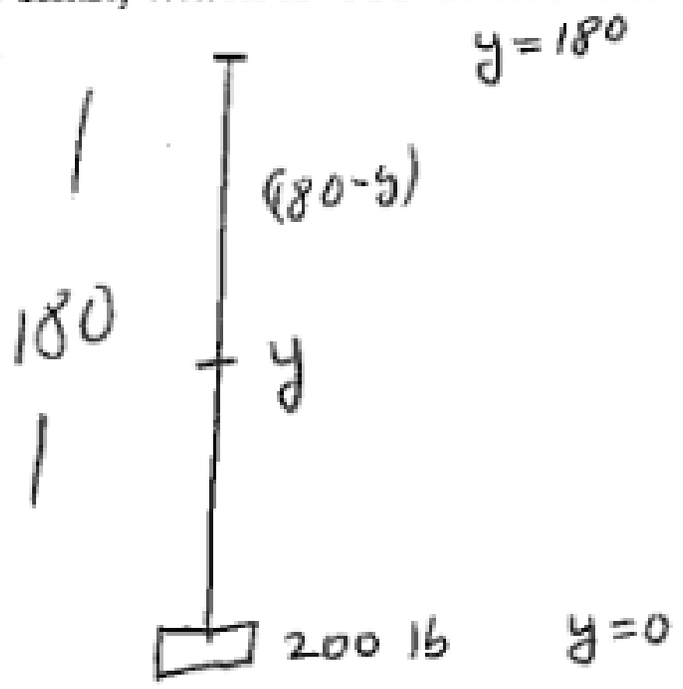
$W(3) = \int_0^3 k x\ dx = 1800$

$k \frac{x^2}{2}\Big|_0^3 = 1800$

$\frac{9k}{2} = 1800 \Rightarrow \boxed{k = 400}$

6) A motor at the top of an elevator shaft is about to lift a 200 lb weight from the ground, using a uniform cable 180 ft long, the whole cable weighing 40 lb. How much **work** in ft-lb does the motor do in lifting the **weight** and the **entire cable** to the **top** of the shaft, which is 180 ft above the ground ?

A) 9400
B) 18500
C) 24800
D) 32400
E) 39600
F) 42200
G) 46400
H) 48600
I) 52400
J) 56800



$\frac{40\ lb}{180\ ft} = \boxed{\frac{2}{9}\ lb/ft}$

$F(y) = 200 + (180 - y)\frac{2}{9}$

$= 200 + 40 - \frac{2}{9} y$

$= 240 - \frac{2}{9} y$

$W = \int_0^{180} 240 - \frac{2}{9} y\ dy$

$= 240\ y - \frac{1}{9} y^2 \Big]_0^{180} = \boxed{39600\ ft\text{-}lb}$

(E)