MATH 132 **EXAM II** **(Solutions)** FALL 2010

1) Evaluate $\int cos^3(x)\,dx$.

$A)\, cos^2(x) + \frac{1}{4}cos^4(x) + C$ $\;B)\; cos(x) + \frac{1}{3}cos^3(x) + C$
$C)\, \frac{1}{2}x - \frac{1}{2}cos(x) + C$ $\;D)\, sin^2(x) - \frac{1}{4}sin^4(x) + C$ $\;E)\; sin(x) - \frac{1}{3}sin^3(x) + C$
$F)\, \frac{1}{2}x + \frac{1}{2}sin(x) + C$ $\;G)\, \frac{1}{4}cos^4(x) + C$ $\;H)\, cos(x) - cos(x)\,sin^2(x) + C$
$I)\, sin(x) - sin(x)\,cos^2(x) + C$ $\;J)\, \frac{1}{2}sin(x) - \frac{1}{3}cos^2(x) + C$

<u>solution</u>: $= \int cos^2(x)\,cos(x)\,dx = \int (1 - sin^2(x))(cos(x))\,dx \;\; (u = sin(x)$
$du = cos(x)dx) = \int 1 - u^2\,du = u - \frac{1}{3}u^3 = sin(x) - \frac{1}{3}sin^3(x) + C \;\; (E)$

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2) What is the form of the integral $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$, after we make the trigonometric substitution, $x = sin(\theta)$.

$A)\int sin(\theta)\,d\theta$ $\;B)\int cos(\theta)\,d\theta$ $\;C)\int sin^2(\theta)\,d\theta$ $\;D)\int cos^2(\theta)\,d\theta$ $\;E)\int sin(\theta)\,cos(\theta)\,d\theta$
$F)\int sin^2(\theta)\,cos(\theta)\,d\theta$ $\;G)\int sin(\theta)\,cos^2(\theta)\,d\theta$ $\;H)\int tan(\theta)\,d\theta$ $\;I)\int sec^2(\theta)\,d\theta$
J) $\int sec(\theta)\,tan(\theta)\,d\theta$

<u>solution</u>: $= \int \frac{sin^2(\theta)}{\sqrt{1-sin^2(\theta)}}\,cos(\theta)d\theta = \int sin^2(\theta)\,d\theta + C \quad (C)$

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3) Use **partial fractions** to evaluate $\int \frac{x+2}{x^2+x}\,dx \;\; (x > 0)$.

$A)\; ln(2x+1)$ $\;B)\; ln(x^2+x)$ $\;C)\; arctan(x)$ $\;D)\; \frac{1}{arctan(x)}$ $\;E)\,(2x+1)^2$
$F)\, ln(\frac{x+1}{x})$ $\;G)\, ln(\frac{x^2}{x+1})$ $\;H)\, ln\,(\frac{2x}{x-1})$ $\;I)\, ln(arctan(x))$ $\;J)\, arctan(lan(x))$

<u>solution</u>: $= \int \frac{2}{x} - \frac{1}{x+1}\,dx = 2ln(x) - ln(x+1) = ln(\frac{x^2}{x+1}) + C \quad (G)$

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4) Approximate $\int_1^2 \frac{1}{x}\,dx$, using the Trapezoidal Rule with $n = 4$, to 4 decimal places.

$A)\, 0.6950$ $\;B)\, 0.6961$ $\;C)\, 0.6953$ $\;D)\, 0.6940$ $\;E)\, 0.6932$ $\;F)\, 0.6922$
$G)\, 0.6917$ $\;H)\, 0.6911$ $\;I)\, 0.6970$ $\;J)\, 0.6900$ $\;\; (\Delta x = \frac{2-1}{4},\;\; \frac{\Delta x}{2} = \frac{1}{8})$

<u>solution</u>: $=\frac{1}{8}(\frac{1}{1} + 2(\frac{1}{5/4}) + 2(\frac{1}{3/2}) + 2(\frac{1}{7/4}) + \frac{1}{2}) = 0.6970238092 \;\; (I)$

5) How large should n be to guarantee that Simpson's Rule of approximation for $\int_0^2 e^{-2x}dx$ is acurate to within 0.0001 ? (recall : $|E_s| \le \frac{K(b-a)^5}{180n^4}$)
$A)\,7\;\; B)\,8\;\; C)\,9\;\; D)\,10\;\; E)\,11\;\; F)\,12\;\; G)\,13\;\; H)\,14\;\; I)\,15\;\; J)\,16$

solution: $f^{(4)}(x) = 16e^{-2x} \le 16\ on\ [0,2] \Rightarrow K = 16$. We want $\frac{16(2^5)}{180n^4} \le 0.0001$.
$Then\ n^4 \ge \frac{16(32)}{180(0.0001)} \;\Rightarrow n \ge (\frac{16(32)}{180(0.0001)})^{1/4} = 12.9867 \Rightarrow n = 14.\ (H)$

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6) Evaluate the improper integral $\int_0^2 \frac{1}{x^2}\, e^{-\frac{1}{x}}dx$, if it is convergent. If not say it is divergent.
$A)\,\frac{1}{\sqrt{e}}\;\; B)\,\frac{1}{e^2}\;\; C)\,\sqrt{e}\;\; D)\,e^2\;\; E)\,\frac{1}{e^{3/2}}\;\; F)\,e^{3/2}\;\; G)\,\frac{1}{e}-1\;\; H)\,\frac{1}{\sqrt{e}}-1$
$I)\,e^2-1\;\; J)\,divergent$

solution: $\int_a^2 \frac{1}{x^2}\, e^{-\frac{1}{x}}dx = e^{-\frac{1}{x}}\Big|_a^2 = e^{-\frac{1}{2}} - e^{-\frac{1}{a}}.\quad \lim_{a\to 0^+} e^{-\frac{1}{2}} - e^{-\frac{1}{a}} = e^{-\frac{1}{2}}.\ (A)$

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7) Find the area of the region enclosed by the curves $y = x^2\ \ and\ \ y = -x^2 + 4x$
$A)\,\frac{1}{3}\;\; B)\,\frac{2}{3}\;\; C)\,\frac{1}{2}\;\; D)\,\frac{3}{2}\;\; E)\,\frac{4}{3}\;\; F)\,\frac{5}{2}\;\; G)\,\frac{5}{3}\;\; H)\,\frac{8}{3}\;\; I)\,\frac{10}{3}\;\; J)\,\frac{13}{3}$

solution: Intersection points are (0,0) and (2,4). $f(x) = -x^2 + 4x$, is upper curve, and $g(x) = x^2$ is the lower curve. Then Area $= \int_0^2 (-x^2 + 4x) - x^2\,dx =$
$\int_0^2 4x - 2x^2\,dx = \frac{8}{3}.\quad (H)$

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11) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve $y = 3 + 2x - x^2$ and the line $x + y = 3$ about the y-axis .

A) π B) $\frac{27\pi}{2}$ $C)\, 2\pi$ $D)\, \frac{28\pi}{3}$ $E)\, 4\pi$ $F)\, \frac{32\pi}{5}$ G) $\frac{30\pi}{7}$
$H)\, 40\pi$ $I)\, \frac{35\pi}{4}$ $J)\, 48\pi$

solution: For cylindrical shell about the y-axis you must describe the region in terms of functions of x. $f(x) = 3 + 2x - x^2$ is the top curve and $g(x) = 3 - x$, is the bottom curve. The intersection points are (0,3) and (3,0). For each x between 0 and 3, the distance to the y-axis is x. Therefore the volume is given by

$2\pi \int_0^3 x((3 + 2x - x^2) - (3 - x))\,dx = 2\pi \int_0^3 3x^2 - x^3\,dx = \frac{27\pi}{2}.$ (B)

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12) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve $y = \sqrt{x}$ and the lines $y = 0,\ x = 1,$ about the line $y = -1$.

A) π B) $\frac{11\pi}{6}$ $C)\, 2\pi$ $D)\, \frac{21\pi}{5}$ $E)\, 4\pi$ $F)\, \frac{32\pi}{15}$ $G)\, \frac{3\pi}{5}$ $H)\, 40\pi$ $I)\, \frac{13\pi}{8}$ $J)\, 48\pi$

solution: About the line $y = -1$, we must use the region between function of y. (1,1) is the intersection point of $x = 1$ and $x = y^2$, with $x = 1$, the top, and $x = y^2$, the bottom. For each y between 0 and 1, the distance to $y = -1$ is $y + 1$.

$V = 2\pi \int_0^1 (y+1)(1 - y^2)\,dy = 2\pi \int_0^1 y + 1 - y^3 - y^2\,dy = \frac{11\pi}{6}.$ (B)

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13) Which of the following integrals gives the length of the curve

$x = t - t^2$, $y = \frac{4}{3} t^{3/2}$, $0 \le t \le 3$?

A) $\int_0^3 \sqrt{3 + 4t^2}\ dt$ $B)\, \int_0^3 \sqrt{1 + 4t^2}\ dt$ $C)\, \int_0^3 \sqrt{3 + 5t^2}\ dt$
$D)\, \int_0^3 \sqrt{1 + 5t^2}\ dt$ $E)\, \int_0^3 \sqrt{3 + 3t^2}\ dt$ $F)\, \int_0^3 \sqrt{1 + 3t^2}\ dt$
$G)\, \int_0^3 \sqrt{3 + t^2}\ dt$ $H)\, \int_0^3 \sqrt{1 + t^2}\ dt$ $I)\, \int_0^3 \sqrt{3 + 2t^2}\ dt$ $J)\, \int_0^3 \sqrt{1 + 2t^2}\ dt$

solution: $\frac{dx}{dt} = 1 - 2t,\ (\frac{dx}{dt})^2 = 1 - 4t + 4t^2,\ \frac{dy}{dt} = 2t^{\frac{1}{2}}\ ,\ (\frac{dy}{dt})^2 = 4t.$

$L = \int_0^3 \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_0^3 \sqrt{1 + 4t^2}\,dt.$ (B)

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14) Find the average value of $y = x^2$ over the interval [1 , 3] .

A) $\frac{10}{3}$ $B)\, \frac{11}{3}$ $C)\, 4$ $D)\, \frac{13}{3}$ $E)\, \frac{14}{3}$ $F)\, 5$ $G)\, \frac{16}{3}$ $H)\, \frac{17}{3}$ $I)\, 6$ $J)\, \frac{19}{3}$

solution: average value $= \frac{1}{2} \int_1^3 x^2\,dx = \frac{13}{3}.$ (D)