

**Math 132**  
**Fall 2007 Exam III**

1. The region in the first quadrant that is bounded above by  $y = 4x - x^2$  and bounded below by  $y = x$  is rotated about the vertical line  $x = -1$ . What is the volume of the resulting solid of revolution?

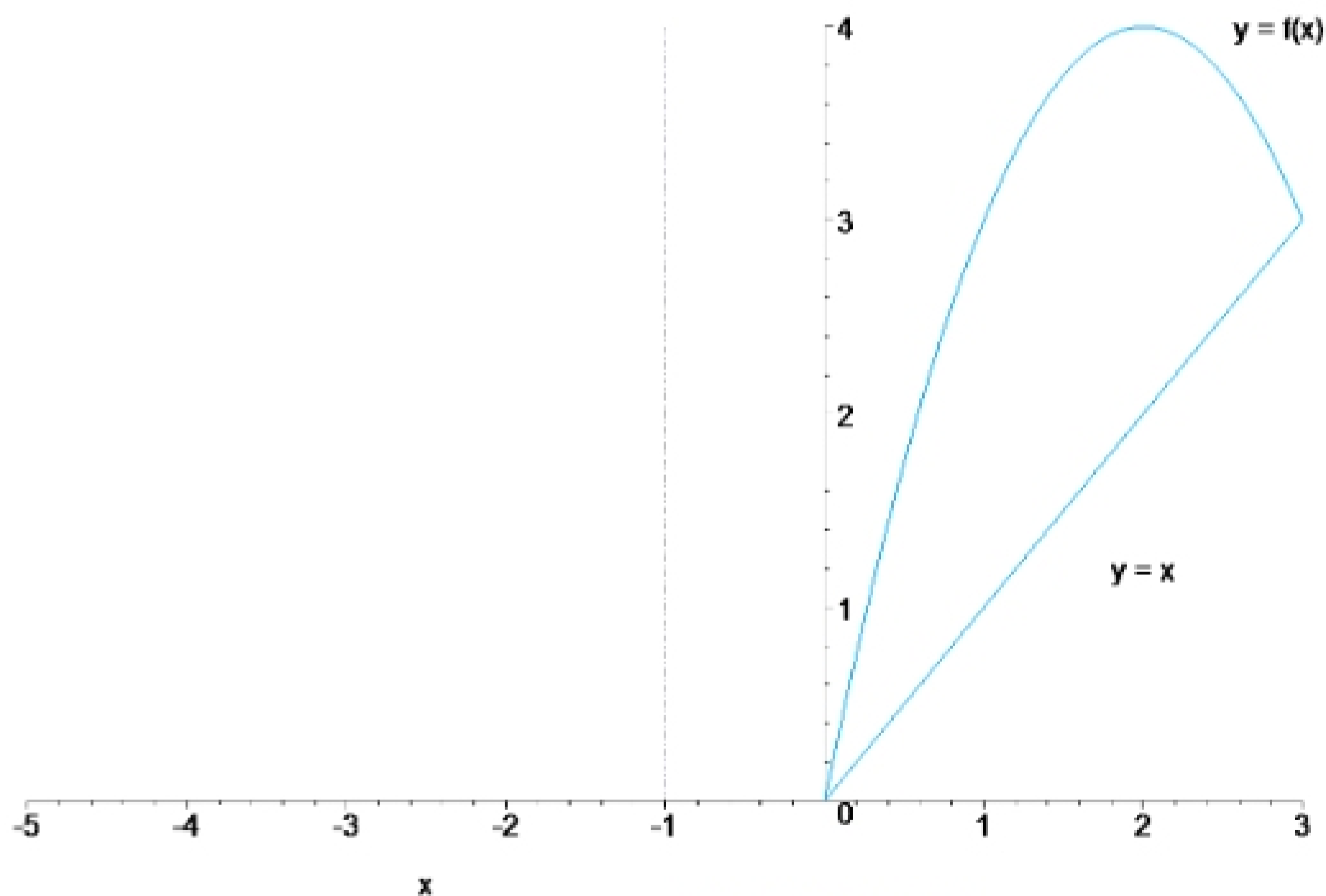
- a)  $\frac{15\pi}{2}$    b)  $16\pi$    c)  $18\pi$    d)  $21\pi$    e)  $\frac{45\pi}{2}$    f)  $20\pi$    g)  $\frac{27\pi}{2}$    h)  $\frac{25\pi}{2}$    i)  $14\pi$   
j)  $15\pi$

**Solution: e**

First we plot the planar region that is to be rotated.

```
> f := x -> 4*x-x^2;   g := x -> x;
      f := x -> 4*x-x^2
      g := x -> x
```

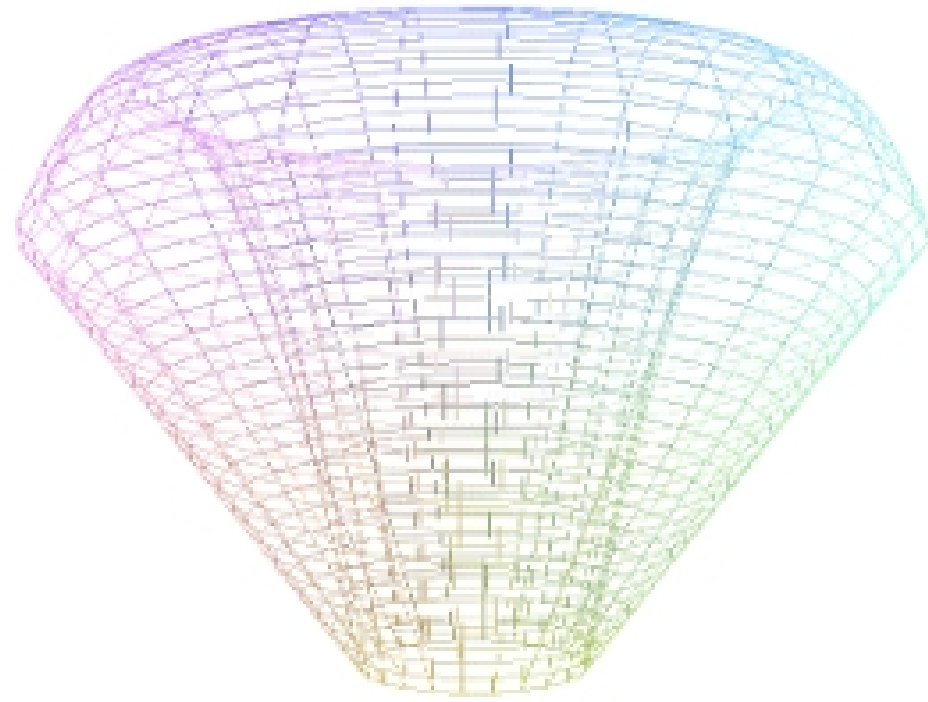
The planar region bounded by  $y = x$  and  $y = 4x - x^2$  is shown below:



Problem 1, Figure 1

When this region is rotated about the specified axis, the following solid results.

### **The Solid of Revolution**

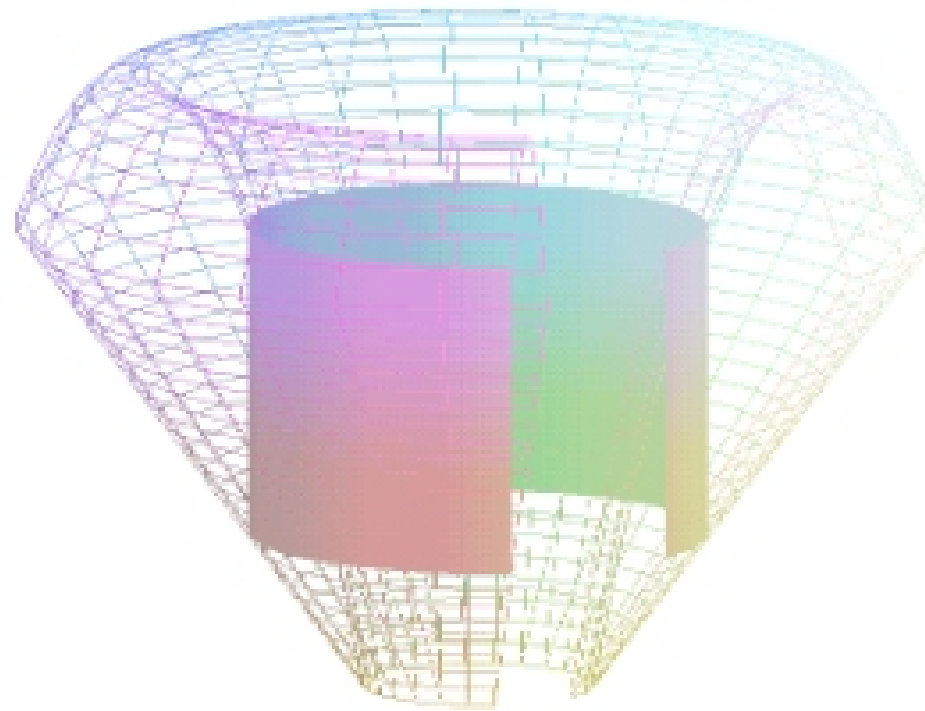


**Problem 1, Figure 2**

## Calculation of the Volume by the Method of Cylindrical Shells

The next figure shows a typical cylindrical shell.

### Cut-away of The Solid of Revolution with One Shell Shown



Problem 1, Figure 3

Here is the calculation of the volume by means of the Method of Cylindrical Shells:

```
> radius_of_shell := x -> x + 1;  
height_of_shell := x -> f(x) - g(x);  
radius_of_shell := x -> x + 1  
height_of_shell := x -> f(x) - g(x)  
> Volume = Int(2*Pi*radius_of_shell(x)*height_of_shell(x), x = 0  
.. 3);
```

$$Volume = \int_0^3 2\pi(x+1)(3x-x^2) dx$$