Math 132, Exam 3, April 11th

All questions carry equal marks. Choose the answer that is closest to the solution (remember this means rounding the number, not just truncating it).

1. Consider the statements:
(I) A necessary condition for $\sum_{n=1}^{\infty} a_n$ to converge is that $\lim_{n\to\infty} a_n = 0$.
(II) A sufficient condition for $\sum_{n=1}^{\infty} a_n$ to converge is that $\lim_{n\to\infty} a_n = 0$.

A. Both I and II are true.
B. I is true, II is false.
C. II is true, I is false.
D. Both I and II are false.

2. The first term of a sequence is $x_1 = 1$. For $n > 1$,

$$x_n = \sum_{k=1}^{n-1} x_k.$$

What is $\ln(x_{100})$?

A. 67.0
B. 67.1
C. 67.2
D. 67.3
E. 67.4
F. 67.5
G. 67.6
H. 67.7
I. 67.8
J. 67.9

$x_1 = 1$

$x_2 = 1$

$x_3 = 1+1 = 2$

$x_4 = 1+1+2 = 4$

$x_5 = 1+1+2+4 = 8$

$\vdots$

$x_n = 2^{n-2}$

$\ln x_{100} = \ln 2^{98} = 98 \ln 2 \approx 67.928$

3. Let $a_n = \frac{1000}{(n+1)(n+2)}$. What is $\sum_{n=1}^{100} a_n$?

A. 490.0
B. 490.1
C. 490.2
D. 490.3
E. 490.4
F. 490.5
G. 490.6
H. 490.7
I. 490.8
J. 490.9

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$\therefore \sum_{n=1}^{100} a_n = 1000 \sum_{n=1}^{100} \frac{1}{n+1} - \frac{1}{n+2}$$

$$= 1000 \left[\frac{1}{2} - \frac{1}{102} \right]$$

$$= 490.196$$