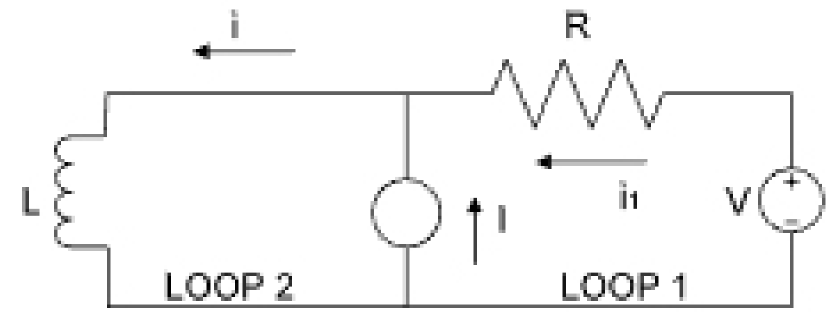
1a)

Consider the electric circuit:



Writing the voltage balance equation for LOOP 1 and LOOP 2,

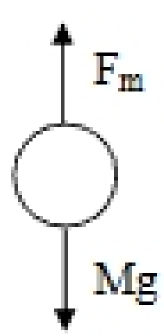
$$-V(t) + Ri_1(t) + L\frac{di(t)}{dt} = 0$$

But,

$$i_1(t) = i(t) - I(t)$$

$$\frac{di(t)}{dt} = \frac{1}{L}V(t) + \frac{R}{L}I(t) - \frac{R}{L}i(t) \qquad (1.1)$$

<u>Free body diagram of the ball</u>



Writing the Newton's second law of motion,

$$\sum F_x = M\ddot{x}(t)$$

$$\Rightarrow M\ddot{x}(t) = Mg - F_m,$$

$$\Rightarrow M\ddot{x}(t) + \frac{i^2(t)}{x(t)} = Mg \qquad (1.2)$$

1b) To obtain equilibrium/operating point(s), set $\frac{di(t)}{dt} = 0;\ \dot{x}(t) = 0;\ \ddot{x}(t) = 0$,

$$V_0 + RI_0 - Ri_0 = 0,$$

$$\Rightarrow i_0 = \frac{V_0 + RI_0}{R}$$

$$\frac{i_0^2}{x_0} = Mg,$$

$$\Rightarrow x_0 = \frac{i_0^2}{Mg}$$

The equilibrium/operating point is not unique as 'i_0' depends on 'V_0' and 'I_0'.

2) The differential equations of a system are

$$x(t)\ddot{y}(t)+\dot{y}(t)\left[4x(t)+\dot{y}(t)\right]+6\sqrt{y(t)}=12,$$
$$\dot{x}(t)+3x(t)=u^2(t)$$

a) To obtain the equilibrium/operating point, equate all derivative terms to zero in the differential equations. It is given that the equilibrium value of u(t) is u_0.

$$6\sqrt{y_0}=12\Rightarrow\sqrt{y_0}=2\Rightarrow y_0=4,$$
$$3x_0=u_0^2$$

Since the value of x_0 depends of the value of u_0, the equilibrium/operating point is not unique.

b) The differential equations are to be linearized about the equilibrium/operating point $E_0=(u_0,\ x_0,\ y_0)=\left(u_0,\ \frac{u_0^2}{3},\ 4\right)$

Define,

$$\Delta x(t)=x(t)-x_0,$$
$$\Delta y(t)=y(t)-y_0,$$
$$\Delta u(t)=u(t)-u_0$$

Apply Taylor series expansion to the first equation,

$$x(t)\ddot{y}(t)\Big|_{E_0}+\frac{\partial}{\partial x}\left(x(t)\ddot{y}(t)\right)\Big|_{E_0}\Delta x(t)+\frac{\partial}{\partial\ddot{y}}\left(x(t)\ddot{y}(t)\right)\Big|_{E_0}\Delta\ddot{y}(t)+4x(t)\dot{y}(t)\Big|_{E_0}+\frac{\partial}{\partial\dot{y}}\left(4x(t)\dot{y}(t)\right)\Big|_{E_0}\Delta\dot{y}(t)$$
$$+\frac{\partial}{\partial x}\left(4x(t)\dot{y}(t)\right)\Big|_{E_0}\Delta x(t)+\dot{y}^2(t)\Big|_{E_0}+\frac{\partial}{\partial\dot{y}}\left(\dot{y}^2(t)\right)\Big|_{E_0}\Delta\dot{y}(t)+6\sqrt{y(t)}\Big|_{E_0}+\frac{\partial}{\partial y}\left(6\sqrt{y(t)}\right)\Big|_{E_0}\Delta y(t)=12,$$
$$\Rightarrow x_0\Delta\ddot{y}(t)+4x_0\Delta\dot{y}(t)+6\sqrt{y_0}+\frac{6}{2\sqrt{y_0}}\Delta y(t)=12,$$
$$\Rightarrow\frac{u_0^2}{3}\Delta\ddot{y}(t)+\frac{4u_0^2}{3}\Delta\dot{y}(t)+\frac{3}{2}\Delta y(t)=0$$