Math 128 Final Exam Fall '09

This exam has 15 questions; indicate your answers on your answer card.

1. For the curve given by $x^3 + x^2 - 2y^2 + 10y - 14 = 0$ find the slope of the line tangent to the curve at the point $(1,2)$.

a. $-9/2$
b. $-7/2$
c. $-5/2$
d. $-3/2$
e. $-1/2$
f. $1/2$
g. $3/2$
h. $5/2$
i. $7/2$
j. $9/2$

$F(x,y) = x^3 + x^2 - 2y^2 + 10y - 14$

$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{3x^2 + 2x}{-4y + 10}$

$x = 1 \quad y = 2$

$= -\frac{5}{-8+10} = -5/2$

2. Find the equation of the plane tangent to the surface $x^3 - 3x^2y^2$ at the point $(x,y,z) = (2,1,-4)$.

$F(x,y)$

a. $z + 24y = 20$
b. $6z + x - y = 30$
c. $z + 3x + 4y = 15$
d. $4z - x - 4y = 3$
e. $z + 2x - 3y = 12$
f. $2z - x = -6$
g. $3z - 3x + 2y = -13$
h. $3z + 3x - y = -3$
i. $12z - 8x + y = 8$
j. $4z - 5x + 6y = 8$

$z - (-4) = F_x(x-2) + F_y(y-1)$

$F_x = 3x^2 - 6xy^2$ → $12 - 6 \cdot 2 \cdot 1 = 0$

$F_y = -6x^2y$ → $-6 \cdot 4 = -24$

so

$z + 4 = 0(x-2) - 24(y-1)$

$z + 4 = -24y + 24$

$z + 24y = 20$

3. There is a point in the region $K, L > 0$ that maximizes the function

$$20K^{1/4}L^{1/2} - 2L - K. \quad = F(K, L)$$

The coordinates of that point are $(K, L) = ?$

a. (125, 3025)

b. (625, 625)

c. (3025, 125)

d. (15125, 25)

e. (2, 1024)

f. (4, 512)

g. (8, 256)

h. (16, 128)

i. (32, 64)

j. (64, 32)

(a) $0 = \frac{\partial F}{\partial K} = 20 \cdot \frac{1}{4} \cdot K^{-3/4} L^{1/2} - 1$

(b) $0 = \frac{\partial F}{\partial L} = 20 \cdot \frac{1}{2} K^{1/4} L^{-1/2} - 2$

From (a) $5K^{-3/4} L^{1/2} = 1$

(b) $5K^{1/4} L^{-1/2} = 1$

set those two equal to each other, find $K = L$ go back to (a) or (b) find $K = L = 5^4$

4. Find and the maximum and minimum values of the function

$$f(x, y) = x^4 + y^4$$

subject to the constraint $x^2 + y^2 = 1$.

a. There is no maximum, no minimum

b. There is no maximum, minimum = 2

c. There is no maximum, minimum = 1/2

d. There is no maximum, minimum = 1

e. Maximum = 2, there is no minimum

f. Maximum = 1, there is no minimum

g. Maximum = 1/2, there is no minimum

h. Maximum = 2, minimum = 1

i. Maximum = 2, minimum = 1/2

j. Maximum = 1, minimum = 1/2

Lagrange multipliers:

$4x^3 - \lambda 2x = 0$

$4y^3 - \lambda 2y = 0$

1st eqn gives

$2x(x^2 - \lambda) = 0$

so $x = 0$ or $x^2 = \lambda$

$x = 0$ gives $y = \pm 1$ 2 points

$x^2 = \lambda$ in the 2nd equation and factor gives $y = 0$ or $x^2 = y^2$ in which $x^2 = y^2 = 1/2$

$x = 0 \quad y = \pm 1$, $y = 0 \quad x = \pm 1$ } all give $f(x, y) = 1$, $x^2 = y^2 = 1/2$ (4 points) } $f(x, y) = 1/2$

5. Find and classify all the stationary points of the function

$$f(x,y) = x^2 - y^3 + y^2 + 6$$

a. Local maximum at $(0,0)$

b. Local maximum at $(0,0)$

c. Saddle point at $(0,0)$

d. Local minimum at $(0,0)$, local maximum at $(0,2)$

e. Local maximum at $(0,0)$, saddle point at $(0,3)$

f. Local minimum at $(0,0)$, local minimum at $(0,3/2)$

g. Saddle point at $(0,0)$, saddle point at $(0,3/2)$

h. Saddle point at $(0,0)$, local maximum at $(0,2/3)$

i. Local minimum at $(0,0)$, saddle point at $(0,2/3)$

j. Local maximum at $(0,0)$, local maximum at $(0,2/3)$

$0 = f_x = 2x$

$0 = f_y = -3y^2 + 2y$

so $x = 0$

and $y(-3y+2) = 0$

$y = 0$ or $-3y + 2 = 0$

$y = 2/3$

$f_{xx} = 2$

$f_{xy} = 0$

$f_{yy} = -6y + 2$

second deriv test gives ~~and~~ ans.

6. Find the coordinates of the point (x,y) that gives the maximum value of $\sqrt{2x} + \sqrt{y}$ subject to the condition $x + y = 1$.

a. $(1,0)$

b. $(3/4, 1/4)$

c. $(2/3, 1/3)$

d. $(1/2, 1/2)$

e. $(1/3, 2/3)$

f. $(2/5, 3/5)$

g. $(1/4, 3/4)$

h. $(1/6, 5/6)$

i. $(5/6, 1/6)$

j. $(0,1)$

There are no questions with inequality constraints. This was going to be one but a typing mistake turned ≤ 1 into $= 1$

Lagrange multipliers

$\frac{1 \cdot 2}{2\sqrt{2x}} - \lambda = 0$

$\frac{1}{2\sqrt{y}} - \lambda = 0$

so $\frac{1}{\sqrt{2x}} = \frac{1}{2\sqrt{y}}$

so $4y = 2x$

$y = x/2$

also $x + y = 1$ so $x = \frac{2}{3}, y = \frac{1}{3}$