1) Use substitution to evaluate $\int_1^2 (2x-1)\cdot\cos(x^2-x)\,dx$.

$A)\,0\quad B)\,0.14\quad C)\,0.23\quad D)\,0.31\quad E)\,0.45\quad F)\,0.53\quad G)\,0.66\quad H)\,0.72\quad I)\,0.84\quad J)\,0.91$

Solution: (u = $x^2 - x$) : $= \int_0^2 cos(u)\,du = sin(u)\,|_0^2 = \sin(2) \sim 0.91$ (J)
(du = (2x-1) dx)

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2) Use integration by parts to evaluate $\int_0^1 x\,e^x\,dx$.

$A)\,0\quad B)\,1\quad C)\,-1\quad D)\,2\quad E)\,-2\quad F)\,e\quad G)\,e^2\quad H)\,\frac{e-1}{2}\quad I)\,\frac{e+1}{4}\quad J)\,e-1$

Solution: (u = x , dv = $e^x\,dx$, du = dx, v = e^x) : = x $e^x|_0^1$ - $\int_0^1 e^x\,dx$ =
x $e^x|_0^1$ - $e^x|_0^1 = 1\ (B)$

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3) Use partial fractions to evaluate $\int \frac{3}{x^2+x-2}\,dx \quad (x>0)$

$A)\,arctan(2x+1)+C\quad B)\,(x^2+x-2)^{-2}+C\quad C)\,ln\,(x^2+x)+C$
$D)\,ln(x-2)+ln(x+2)+C\quad E)\,ln\,(\frac{x-1}{x+2})+C\quad F)\,arcsin(2x+1)+C$
$G)\,\sqrt{2x+1}\ +C\quad H)\,\sqrt{x-1}-\sqrt{x+2}+C\quad I)\,x^2-\sqrt{x}+C$

Solution: $\frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \Rightarrow B = -A,\ 2A - B = 3 \Rightarrow$
$A = 1,\ B = -1.$ So $\int \frac{3}{x^2+x-2}\,dx = \int \frac{1}{x-1} - \frac{1}{x+2}\,dx = ln\,(\frac{x-1}{x+2}) + C\,(E)$

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4) Find what becomes of the integral $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$, when you make the substitution x = 3 sin(θ) , $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

$A)\,3\int sin(\theta)\,d\theta\quad B)\,3\int cos(\theta)\,d\theta\quad C)\,\int \frac{9}{sin(\theta)}\,d\theta\quad D)\,\int \frac{9}{cos(\theta)}\,d\theta\quad E)\,3\int sec^2(\theta)\,d\theta$
$F)\,3\int csc^2(\theta)\,d\theta\quad G)\,9\int sin^2(\theta)\,d\theta\quad H)\,9\int cos^2(\theta)\,d\theta\quad I)\,\int\sqrt{9-sin^2(\theta)}\,d\theta$
$J)\,\int\sqrt{9-cos^2(\theta)}\,d\theta$

Solution: (x = 3 $sin(\theta)$, dx = 3 $\cos(\theta)\,d\theta$) $= \int \frac{9\,sin^2(\theta)}{\sqrt{9-9sin^2(\theta)}}\,3\cos(\theta)\,d\theta =$
$\int \frac{9\,sin^2(\theta)}{3\,cos(\theta)}\,3\cos(\theta)\,d\theta = 9\int sin^2(\theta)\,d\theta.\ (G)$

5) Find the area of the region enclosed by the parabola $y = 2 - x^2$ *and* the line $y = -x$.

$A)\ 1\ \ B)\ \frac{1}{2}\ \ C)\ \frac{3}{2}\ \ D)\ \frac{4}{3}\ \ E)\ \frac{7}{5}\ \ F)\ \frac{5}{3}\ \ G)\ \frac{11}{5}\ \ H)\ \frac{9}{2}\ \ I)\ \frac{14}{3}\ \ J)\ \frac{13}{5}$

Solution: Intersection when $2 - x^2 = -x \Rightarrow x^2 - x - 2 = 0 \Rightarrow$ $x = -1, 2$. $A = \int_{-1}^{2} (2 - x^2 + x)\,dx = 2x - \frac{x^3}{3} + \frac{x^2}{2} \Big|_{-1}^{2} =$ $(4 - \frac{8}{3} + 2) - (-2 + \frac{1}{3} + \frac{1}{2}) = \frac{9}{2}$ (H)

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6) Find the **volume** of the solid you get *by* **revolving** the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \le y \le 4$, **about the y-axis .**

A) $\frac{\pi}{4}$ $B)\ \frac{3\pi}{4}$ $C)\ \pi$ $D)\ \frac{5\pi}{4}$ $E)\ \frac{7\pi}{4}$ $F)\ \frac{9\pi}{2}$ $G)\ 2\pi$ $H)\ \frac{9\pi}{4}$ $I)\ 3\pi$ $J)\ \frac{4\pi}{3}$

Solution: For any y between 1 and 4, the cross section will be a circle with the radius being the x-coordinate on the curve , which means $r = x = \frac{2}{y}$, *i.e.* $A(y) = \pi\ \frac{4}{y^2}$.
Then volume is $V = \pi \int_1^4 \frac{4}{y^2}\,dy = -\frac{4}{y}\Big|_1^4 = \pi(4 - 1) = 3\pi$ (I)

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7) The base of the solid **S** is in the x-y plane, it is the area inside the **triangle** having vertices (0,0), (1,0) and (0,1). If the cross-section of the solid, for each $0 \le y \le 1$, is a semicircle whose diameter is that part of the cross-section that lies in the x-y plane, then **find the volume of that solid.**

A) $\frac{\pi}{24}$ B) $\frac{\pi}{12}$ $C)\ \frac{\pi}{6}$ $D)\ \frac{\pi}{3}$ $E)\ \frac{\pi}{2}$ $F)\ \frac{7\pi}{24}$ $G)\ \frac{5\pi}{12}$ $H)\ \frac{7\pi}{6}$ $I)\ \frac{4\pi}{3}$ $J)\ \frac{3\pi}{2}$

Solution: The line connecting (1,0) and (0,1) has equation $y = 1 - x$. So for each $0 \le y \le 1$, the diameter of the semicircle has length $x = 1 - y$. That gives us that $A(y) = \frac{1}{2}\pi(\frac{1-y}{2})^2$, half the area inside the full circle, with radius $\frac{1-y}{2}$. Then the volume is $\frac{\pi}{8}\int_0^1 (1 - 2y + y^2)\,dy = \frac{\pi}{8}(y - y^2 + \frac{y^3}{3})\Big|_0^1 = \frac{\pi}{24}$. (A)

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8) Find the length of the curve with parametric equations

$x = t^3$, $y = \frac{3}{2}t^2$, $0 \le t \le \sqrt{3}$.

A) 1 $B)$ 2 $C)$ 3 $D)$ 4 $E)$ 5 $F)$ 6 $G)$ 7 $H)$ 8 $I)$ 9 $J)$ 10

Solution: $(\frac{dx}{dt})^2 = (3t^2)^2$ and $(\frac{dy}{dt})^2 = (3t)^2$. So the length is
$L = \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\,dt = \int_0^{\sqrt{3}} 3t\sqrt{t^2+1}\,dt$ $(u = t^2 + 1,\ du = 2t\,dt) =$
$\int_1^4 \frac{3}{2}u^{\frac{1}{2}}\,du = u^{\frac{3}{2}}\Big|_1^4 = 7$ (G)

9) Determine whether the improper integral $\int_{-\infty}^{1} \frac{2}{1+x^2}\, dx$ converges, and if so determine its value .

A) 0 B) 2 C) 4 $D)\,6$ $E)\,8$ $F)\,\frac{\pi}{6}$ $G)\,\frac{\pi}{4}$ $H)\,\frac{2\pi}{3}$ $I)\,\frac{3\pi}{2}$ $J)\,diverges$

Solution: $\int_{-\infty}^{1} \frac{2}{1+x^2}\, dx = 2\,arctan(x)\,\Big|_{-\infty}^{1} = 2(\frac{\pi}{4} - (-\frac{\pi}{2}) = \frac{3\pi}{2}$. (I)

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10) A spring has a natural length of **4 ft.**, and the work needed to stretch it to **12 ft** is **64 ft-lb.** How much **work (in ft-lb)** is needed to compress it from **4 ft** to **2 ft** ?

A) 2 B) 4 C) 6 D) 8 E) 10 F) 12 G) 14 H) 16 I) 18 J) 20

Solution: In general the force F(x) = k x , where k is a constant and x is size of stretch or size of compression. We are given that W = 64 ft-lb for a stretch of 8 ft so we have $64 = \int_0^8 k\,x\,dx \Rightarrow 64 = k\,\frac{x^2}{2}\Big|_0^8 = 32\,k \Rightarrow k = 2$. So the work to to compress the spring from 0 to 2 is given by $W = \int_0^2 2\,x\,dx = x^2\,\Big|_0^2 = 4$ ft-lb . (B)

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11) Find the solution to the differential equation $\frac{dy}{dx} = 2x^2\sqrt{y}$, with initial condition $y(0) = 0$.

$A)\,y = (x+2)^{-1}$ $B)\,y = 3x^2$ $C)\,y = (x+2)^{-2}$ $D)\,y = \frac{x^4}{3}$ $E)\,y = 4\,x^3$
$F)\,y = \frac{x^4}{2}$ $G)\,y = \frac{x^5}{8}$ $H)\,y = \frac{x^6}{9}$ $I)\,y = \frac{x^6}{6}$ $J)\,y = \frac{x^6}{3}$

Solution: $\int y^{-\frac{1}{2}}\,dy = \int 2x^2\,dx \Rightarrow 2\,y^{\frac{1}{2}} = \frac{2}{3}\,x^3 + C$. The initial condition

gives us that C = 0, so $y^{\frac{1}{2}} = \frac{x^3}{3} \Rightarrow y = \frac{x^6}{9}$. (H)

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12) Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{2^n - 1}{3^n}$.

$A)\,\frac{1}{2}$ $B)\,\frac{3}{2}$ $C)\,\frac{5}{2}$ $D)\,\frac{1}{3}$ $E)\,\frac{2}{3}$ $F)\,\frac{4}{3}$ $G)\,\frac{1}{4}$ $H)\,\frac{3}{4}$ $I)\,\frac{5}{3}$ $J)\ series\,diverges$

Solution: By rules of series $\sum_{n=1}^{\infty} \frac{2^n-1}{3^n} = \sum_{n=1}^{\infty} \frac{2^n}{3^n} - \sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{2^n}{3^n} - \sum_{n=1}^{\infty} \frac{1}{3^n}$.
Both are geometric series. The first has $a = \frac{2}{3}$, $r = \frac{2}{3}$, and the second is $a = \frac{1}{3}$, $r = \frac{1}{3}$.

The the sum is $\frac{\frac{2}{3}}{1-\frac{2}{3}} - \frac{\frac{1}{3}}{1-\frac{1}{3}} = 2 - \frac{1}{2} = \frac{3}{2}$. (B)