

1) Evaluate $\int_0^{\frac{1}{2}} e^{\sin(x)} \cdot \cos(x) dx$.

A) 0 B) 1 C) e *D) $e - 1$ E) $e^2 - 1$ F) $\frac{e^2-1}{2}$ G) $e^{\frac{1}{2}-1}$ H) $e^1 - 1$ I) $e^2 - 1$

Solutions: ($u = \sin(x) du = \cos(x) dx$) $= \int_0^1 e^u du = e^u \Big|_0^1 = e - 1$ (D)

2) Evaluate $\int_0^1 x \sin(2x) dx$.

A) 0 B) $\frac{\sqrt{2}}{2}$ C) $1 - 1$ *D) $\frac{-1}{2}$ E) 21 F) $\frac{\sqrt{2}}{2} - 1$ G) -21 H) 1 I) $\frac{1}{4}$ J) $21 - 1$

Solutions: $u = x$, $dv = \sin(2x) dx$, $du = dx$, $v = -\frac{1}{2} \cos(2x) dx$.

$\int x \sin(2x) dx = -\frac{1}{2} x \cos(2x) + \int \frac{\cos(2x)}{2} dx = -\frac{1}{2} x \cos(2x) + \frac{\sin(2x)}{4}$.

So $\int_0^1 x \sin(2x) dx = -\frac{1}{2} x \cos(2x) + \frac{\sin(2x)}{4} \Big|_0^1 = \frac{-1}{2}$ (D)

3) Use partial fractions to evaluate $\int_1^2 \frac{x+4}{x(x+1)} dx$.

A) 1.24 B) 1.38 C) 1.46 *D) 1.56 E) 1.69 F) 1.75 G) 1.80 H) 1.91 I) 2.14

Solutions: $\frac{x+4}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{(A+B)x+A}{x(x+1)} \Rightarrow A = 4$ and $B = -3$.

Then $\int_1^2 \frac{x+4}{x(x+1)} dx = \int_1^2 \frac{4}{x} - \frac{3}{x+1} dx = 4 \ln(x) - 3 \ln(x+1) \Big|_1^2 =$

$4 \ln(2) - 3 \ln(3) + 3 \ln(2) = 7 \ln(2) - 3 \ln(3) = 1.556 \sim 1.56$ (D)

4) Find what becomes of the integral $\int \frac{x^3}{4+x^2} dx$, when you make the substitution $x = 2 \tan(\theta)$, $-\frac{1}{2} < \theta < \frac{1}{2}$.

*A) $4 \int \tan^3(\theta) d\theta$ B) $\frac{1}{4} \int \sec^3(\theta) d\theta$ C) $\frac{1}{2} \int \sec(2\theta) d\theta$ D) $4 \int \cot^2(\theta) d\theta$

E) $\frac{1}{2} \int \csc^2(\theta) d\theta$ F) $\frac{1}{4} \int \sec^3(\theta) - 1 d\theta$ G) $2 \int \tan^2(\theta) d\theta$

H) $\int 1 - \tan^3(\theta) d\theta$ I) $2 \int 1 + \cot^2(\theta) d\theta$ J) $\int 1 + \csc^3(\theta) d\theta$

Solution: Then $dx = 2 \sec^2(\theta) d\theta$. $\int \frac{x^3}{4+x^2} dx \Rightarrow \int \frac{8 \tan^3(\theta)}{4+4 \tan^2(\theta)} 2 \sec^2(\theta) d\theta \Rightarrow$

$4 \int \frac{\tan^3(\theta) \sec^2(\theta)}{2 \sec^2(\theta)} d\theta = 4 \int \tan^3(\theta) d\theta$ (A)

2.

5) Find the area of the region(s) enclosed by $f(x) = (x - 1)^2$ and $f(x) = 1 - x$.

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) 1 D) $\frac{5}{4}$ E) 2 *F) $\frac{1}{6}$ G) $\frac{5}{6}$ H) $\frac{7}{6}$ I) 3 J) $\frac{5}{3}$

Solutions: $(x - 1)^2 = 1 - x$ for $x = 0$ and $x = 1$. On $[0, 1]$, line is above.
So $A = \int_0^1 (1 - x) - (x - 1)^2 dx = \int_0^1 x - x^2 dx = \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1 = \frac{1}{6}$ (F)

6) Find the volume of the solid you get by **revolving** the region enclosed by the curve $x = \sqrt{y}$, the lines $x = 0$ and $y = 4$, **about the B-axis**.

A) 12.61 B) 14.41 C) 17.21 D) 19.81 *E) 25.61 F) 28.41
G) 31.41 H) 33.81 I) 35.61 J) 38.21

Solution: Region of revolution in x-y plane is enclosed by $y = x^2$ and $y = 4$ with $0 \leq x \leq 2$. By disc method $V = \pi \int_0^2 16 - x^4 dx = \pi (16x - \frac{x^5}{5}) \Big|_0^2 = 25.6\pi$ (E)

7) Suppose S is the solid whose base in the region bounded by the parabola $y = 4 - x^2$ and the line $y = 0$ and whose cross sections perpendicular to the y-axis are squares. For each y between 0 and 4 **find a formula** for the cross section A(y).

A) y^2 B) $4y^2$ C) $4y$ D) $2y^2$ E) $8 - 2y$ F) $y^2 + 4$ G) $4y + 2$ H) $12 - y^2$
I) $2y + 4$ *J) $16 - 4y$

Solution: The region has its top at the vertex of $y = 4 - x^2$, (0, 4), and its bottom on the x-axis, so the interval is $0 \leq y \leq 4$. For each y the side of the square has length $2x$, where x is the x-coordinate of a point (x, y) on $y = 4 - x^2$. Therefore $A(y) = 4x^2 = 4(4 - y) = 16 - 4y$ (J)

8) Calculate the arc length of the curve $y = \frac{2}{3}x^{3/2}$, $0 \leq x \leq 3$.

A) $\frac{1}{2}$ B) $\frac{5}{3}$ C) 2 D) $\sqrt{5}$ E) $\sqrt{2}$ F) $\frac{7}{3}$ G) $\frac{3}{2}$ H) $\sqrt{\frac{1}{2}}$ *I) $\frac{14}{3}$ J) 4

Solution: $\frac{dy}{dx} = x^{1/2}$, $(\frac{dy}{dx})^2 = x$, so length is $l = \int_0^3 \sqrt{1 + x} dx$
($u = 1 + x$, $du = dx$) $= \int_1^4 u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_1^4 = \frac{2}{3} (8 - 1) = \frac{14}{3}$ (I)

3.

9) Determine whether the improper integral $\int_0^8 \frac{1}{\sqrt[3]{8-x}} dx$ converges and if so determine its value .

A) 0 B) 2 C) $\frac{4}{3}$ D) 4 E) $\sqrt[3]{7}$ F) $6\frac{2}{3}$ *G) 6 H) 8 I) $\frac{1}{2}$ J) *diverges*

Solution: $\int (8-x)^{-1/3} dx = -\frac{3}{2}(8-x)^{2/3} \Rightarrow \int_0^8 \frac{1}{\sqrt[3]{8-x}} dx =$
 $-\frac{3}{2}(8-x)^{2/3} \Big|_0^8 = 0 - (-\frac{3}{2} 8^{2/3}) = 6$ (G)

10) A force of 20 N is required to hold a spring stretched 5 m beyond its natural length. How much work is done in stretching it from 5 m to 10 m beyond its natural length ?

A) 50 J B) 90 J C) 110 J D) 130 J *E) 150 J F) 170 J G) 190 J H) 210 J
 I) 230 J J) 250 J

Solution: Given $20 = 5k \Rightarrow k = 4$ and $F(x) = 4x$. Therefore $W = \int_5^{10} 4x dx =$
 $2x^2 \Big|_5^{10} = 150 J$ (E)

11) Find the solution to the initial value differential equation $\frac{dy}{dt} = -y^2$, $y(0) = \frac{1}{2}$.

*A) $y = (t+2)^{-1}$ B) $y = t + \frac{1}{2}$ C) $y = (t + \sqrt{2})^{-2}$ D) $y = (t + \frac{1}{\sqrt{2}})^2$
 E) $y = (t+1)^2 - \frac{1}{2}$ F) $y = (t+2)^{-1} - \frac{1}{2}$ G) $y = (t+2)^{-2} - \frac{1}{2}$
 H) $y = e^{t-\ln(t)}$ I) $y = \sin(2t) + \frac{1}{2}$ J) $y = \cos(t) - \frac{1}{2}$

Solution : $\int -\frac{1}{y^2} dy = \int dt + C$. Then $\frac{1}{y} = t + C$. $y(0) = \frac{1}{2} \Rightarrow C = 2$.
 So we get $y = (t+2)^{-1}$ (A)

12) Find the sum of the geometric series $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \frac{48}{625} - \dots$

A) $\frac{15}{4}$ *B) $\frac{15}{7}$ C) $\frac{13}{5}$ D) $\frac{9}{5}$ E) $\frac{11}{25}$ F) $\frac{25}{12}$ G) $\frac{25}{3}$ H) $\frac{125}{27}$ I) $\frac{125}{3}$ J) $\frac{25}{17}$

Solution: $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \frac{48}{625} - \dots = 3(1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \dots)$.
 We have geometric series with $c = 3$ and $r = -\frac{2}{5}$. Therefore it converges to

$$\frac{3}{1+\frac{2}{5}} = \frac{15}{7} \quad (B)$$